Math Grab Bag

A Workshop by GED Testing Service®
Session Objectives

• Discuss the challenges students have with key concepts in math
• Focus on the foundational “must haves” in mathematical reasoning
• Review tips and strategies for helping students build consistency in math
• Share resources

It Really Isn’t Genetic

35^2 \times 469y^3 = 17^2 \div 17x^2
15[402]
x^2 + y^3 = 472 \div 2^3 + y
I \times x^2 + 7.12

"I HAD MY DOCTOR DO A DNA BLOOD ANALYSIS. AS I SUSPECTED, I'M MISSING THE MATH GENE."

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Math involves…
- Memory
- Language
- Sequencing
- Spatial ordering
- Critical thinking
- Good problem-solving strategies
- Number sense
- Reasoning
- Making connections

In the Classroom, We Often…
- Make assumptions about the presence or absence of foundational skills
- Introduce new concepts too rapidly
- Insufficiently support explanations and activities
- Provide insufficient practice
- Focus on facts versus concepts
- Limit access to manipulatives
- Limit connection of skills to real-life situations
What’s the Problem?

Students at Level 1

• Have limited, but developing proficiency
• Perform some math skills at a basic level, such as
  • Putting fractions/decimals on a number line
  • Solving two-step arithmetic problems
• Are inconsistent in the application of skills
• Lack fluency in basic operations and mathematical properties

Performance Level Descriptors (PLDs)

• Helpful tool for the classroom
• Explain in detail the skills students need to demonstrate to pass the test
How to Use PLDs in the Classroom

Use PLDs to:

Tip 1: Assess student’s current skill level
Tip 2: Determine when students are ready to test
Tip 3: Shape learning activities
Tip 4: Add perspective to lesson plans

Our Students Need…

A Balanced Mathematics Program

Practice makes PERFECT

Fluency

CONSISTENCY IS
Understanding Skills Students Have

<table>
<thead>
<tr>
<th>Low Intermediate Basic Education (4-5.9 GLE)</th>
<th>High Intermediate Basic Education (6-8.9 GLE)</th>
<th>Low Adult Secondary Education (9-10.9 GLE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students can perform with high accuracy all four basic math operations using whole numbers up to three digits and can identify and use all basic mathematical symbols.</td>
<td>Students can perform all four basic math operations with whole numbers and fractions; can determine correct math operations for solving narrative math problems and can convert fractions to decimals and decimals to fractions; and can perform basic operations on fractions.</td>
<td>Students can perform all basic math functions with whole numbers, decimals, and fractions; can interpret and solve simple algebraic equations, tables, and graphs and can develop own tables and graphs; and can use math in business transactions.</td>
</tr>
</tbody>
</table>

Best Practice for Building Skills

![Diagram of shapes and fractions]

(eq) $\frac{1}{2} \times 1 = \frac{1}{2}$
C-R-A – Essential for Understanding

<table>
<thead>
<tr>
<th>Concrete</th>
<th>Representational</th>
<th>Abstract</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students manipulate hands-on, concrete materials</td>
<td>Students draw and observe diagrams, or watch the teacher touching and moving hands-on materials</td>
<td>Students use numbers and mathematical symbols</td>
</tr>
</tbody>
</table>

What are the Foundational “Must-Haves” in Mathematical Reasoning
Building Number Sense

Use a Number Line

Students with Number Sense...

- Think and reason flexibly with numbers
- Use numbers to solve problems
- Spot unreasonable answers
- Understand how to put numbers together and take them apart
- Understand number relationships
But they are just numbers…

<table>
<thead>
<tr>
<th>Type of Number</th>
<th>Quick Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting Numbers</td>
<td>{1, 2, 3, \ldots}</td>
</tr>
<tr>
<td>Whole Numbers</td>
<td>{0, 1, 2, 3, \ldots}</td>
</tr>
<tr>
<td>Integers</td>
<td>{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots}</td>
</tr>
<tr>
<td>Rational Numbers</td>
<td>\frac{p}{q} – p and q are integers, q is not zero</td>
</tr>
<tr>
<td>Irrational Numbers</td>
<td>\pi \approx 3.14159265358979323856… cannot be written as a simple fraction</td>
</tr>
<tr>
<td></td>
<td>\sqrt{3}, \sqrt{99}</td>
</tr>
<tr>
<td>Real Numbers</td>
<td>Rational and Irrational</td>
</tr>
</tbody>
</table>

Defining Numbers

Choose a Number Set:
- Counting
- Integers
- Rationals
- Real

Rational numbers are numbers that can be written as the ratio of two integers.

Example: 5/2, 2.5, -1/4 (-0.25), etc.

https://unctv.pbslearningmedia.org/resource/mgbh.math.ns.numbline/building-a-number-line/#.WU1B-IWcHnM
The Number Line

- Provides a model for basic operations for all rational numbers
- Is a spatial object
- Allows students to situate themselves spatially in mathematics
- Permits students to conceptualize mathematics

Just a Sample

Julia wants to spend $100 or less ordering shirts from an online company. The company charges a $5 shipping fee for any order. The inequality \( S + 5 \leq 100 \) represents the number of shirts, \( n \), Julia can order from the online company. Graph all possible numbers of shirts that Julia can buy.

Click on the number line to plot the point(s).

(NOTE: To remove a point, place the arrow over the point and click the left mouse button.)
Fractions

Students need to learn to locate a fraction on a number line.

Use the number line so students understand that fractions are numbers and not just part of a pizza.

Fraction Tiles

A linear model gives an overview and shows relationships.
Fraction Tiles

What is more, 1/4 or 1/3? What is more, 1/9 or 1/10?

Can students identify the problem?

a) $\frac{2}{5} + \frac{1}{3} = \frac{3}{8}$

b) $\frac{2}{7} + \frac{1}{3} = \frac{2}{21} + \frac{1}{21} = \frac{3}{21}$

c) $\frac{3}{8} + \frac{1}{4} = \frac{9}{32} + \frac{8}{32} = \frac{17}{32}$

d) $\frac{8}{9} + \frac{1}{4} = \frac{32}{36} + \frac{9}{36} = \frac{41}{36}$

e) $\frac{7}{12} + \frac{5}{24} = \frac{14}{24} + \frac{5}{24} = \frac{17}{24}$

f) $\frac{6}{1} - \frac{4}{3} = 6 - \frac{4}{3} = \frac{2}{3} - \frac{4}{6} = \frac{2}{1}$

g) $\frac{5}{2} + \frac{1}{3} = \frac{5}{4} + \frac{1}{15} = \frac{6}{20} = \frac{3}{20}$

h) $\frac{1}{2} \times \frac{1}{3} = \frac{5}{10} \times \frac{2}{10} = \frac{10}{100} = \frac{1}{10}$

i) $\frac{2}{1} + \frac{1}{2} = \frac{5}{4} - \frac{7}{8} = \frac{35}{32} = \frac{3}{8}$

j) $\frac{17}{20} - \frac{4}{3} = \frac{17}{20} - \frac{8}{20} = \frac{17}{20} - \frac{8}{20} = \frac{1}{8} = \frac{4}{16} = \frac{1}{40}$

"Five out of four people have trouble with fractions."

-- Steven Wright
Can Students Use a Number Line?

The fractions $\frac{3}{4}$ and $\frac{2}{3}$ are pictured with number lines below:

\[
\begin{array}{c}
\frac{3}{4} \\
\frac{2}{3}
\end{array}
\]

Correct. The first fraction is greater than the second fraction.

\[
\frac{3}{4} > \frac{2}{3}
\]

Absolute Value

Absolute Value means how far a number is from 0.

- Remove any negative sign and think of all numbers as positive
- Recognize symbol used to represent absolute value

\[| -5 | = 5\]
\[| 7 | = 7\]

"6" is 6 away from zero, and "-6" is also 6 away from zero.

So the absolute value of 6 is 6, and the absolute value of -6 is also 6
Inequalities

An inequality is a math statement that defines a range of values.

Jeffrey runs at least two miles every day.

On November 28, the temperature in North Pole, Alaska is expected to be greater than $-4^\circ$ and less than $9^\circ$.

$T < 6$


Operations with Positive and Negative Integers

$6 + (-2) = 4$

$(-8) - (-3) = -5$
Data Displays

![Histogram and Box Plot](image)

Essential Skills for Box Plots

- Arrange data in order
- Find **median** for all data
- Find **median** for lower quartile
- Find **median** for upper quartile
- Find the **range** of the extreme values
Number Lines and the NFL

https://unctv.pbslearningmedia.org/resource/mket-math-ns-ratnumb/football/#.WU0tDGgrKUk

Resources

Helping with Math - Number Line Generator
http://www.helpingwithmath.com/printables/others/NumberLineGenerator01.htm

Math Warehouse – Number Line Graph Maker
http://www.mathwarehouse.com/number-lines/number-line-maker.php

Math is Fun – Number Lines (Inequalities, Operations, etc.)
http://www.mathsisfun.com/number-line.html

Annenberg Learner – Building the Number Line
http://www.learner.org/courses/learningmath/number/session1/part_c/index.html
Order Matters

The Importance of Understanding the Order of Operations

Why Bother?

Here is your problem: 4 + 2×3 =
Is the answer 18 or 10?

• Avoid confusion in how problems are solved
• Set up rules of precedence or rank of operations
• Is critical to simplifying and solving different algebra problems
Answer the Why

https://www.khanacademy.org/math/pre-algebra/pre-algebra-arith-prop/pre-algebra-order-of-operations/v/introduction-to-order-of-operations

Get Rid of Misconceptions about Order of Operations

Misconception 1 - All multiplication should happen before division.

<table>
<thead>
<tr>
<th>Incorrect</th>
<th>Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>$12 \div 3 \times 4$</td>
<td>$12 \div (3 \times 4)$</td>
</tr>
<tr>
<td>$12 \div 12$</td>
<td>$4 \times 4$</td>
</tr>
<tr>
<td>$1$</td>
<td>$16$</td>
</tr>
</tbody>
</table>

Misconception 2 – All addition comes before subtraction.

<table>
<thead>
<tr>
<th>Incorrect</th>
<th>Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4 + 10 - 5 + 8$</td>
<td>$4 + (10 - 5) + 8$</td>
</tr>
<tr>
<td>$24 - 10 + 2$</td>
<td>$14 - 5 + 8$</td>
</tr>
<tr>
<td>$3$</td>
<td>$9 + 8$</td>
</tr>
<tr>
<td></td>
<td>$17$</td>
</tr>
</tbody>
</table>
Essential Understanding

1. **Parentheses and Brackets**
   from the inside out

2. **Exponents**
   of numbers or parentheses

3. **Multiplication and Division**
   in the order they appear

4. **Addition and Subtraction**
   in the order they appear

Your Turn

What is the value of $6 \div 3 + 4 \times 2$?

So we do the division and multiplication before any addition or subtraction:

$$6 \div 3 + 4 \times 2 = 2 + 4 \times 2 = 2 + 8 = 10$$
Your Turn

What is the value of $6 \times 4 - 12 \div 3 - 8$?

We do $6 \times 4$ and $12 \div 3$ first, then the subtractions:

$$
6 \times 4 - 12 \div 3 - 8
$$

$$
= 24 - 12 \div 3 - 8
$$

$$
= 24 - 4 - 8
$$

$$
= 20 - 8
$$

$$
= 12
$$

Your Turn

What is the value of $20 - (3 \times 2^3 - 5)$?

We start inside the Parentheses, and then use "Exponents" first:

$$
20 - (3 \times 2^3 - 5) = 20 - (3 \times 8 - 5)
$$

[Because $2^3$ means $2 \times 2 \times 2 = 8$, not $2 \times 3 = 6$]

Next Multiply:

$$
20 - (3 \times 8 - 5) = 20 - (24 - 5)
$$

Next Subtract (still working inside the parentheses):

$$
20 - (24 - 5) = 20 - 19
$$

Now the Parentheses are completed, the last operation is Subtract:

$$
20 - 19 = 1
$$
Your Turn

What is the value of \((12 \div 3 + 4) - (4^2 - 6 \times 2)\)?

\[
\frac{12}{3} + 4 - (4^2 - 6 \times 2) \\
= 4 + 4 - (4^2 - 6 \times 2) \\
= 8 - (4^2 - 6 \times 2) \\
= 8 - (16 - 12) \\
= 8 - 4 \\
= 4
\]

What’s Your Sign?

In the equation below, replace each question mark with one of the four mathematical signs: +, -, ×, or ÷. Each sign can be used only once. Fill in the blanks to solve the equation. (Hint: the first sign is +.)

\[
7 \, ? \, 5 \, ? \, 4 \, ? \, 7 \, ? \, 6 = 15
\]

What is the first step that you need to take in order to solve the equation? Write your answer in the question box.

\[
(7 + 5) \div 4 \times 7 - 6 = 15
\]
Percent Change
From Shopping to Identifying Trends
Do your students know the vocabulary?

Ratio – a comparison between two different values
Percent of change – ratio of the amount of change to the original amount
Percent increase – how much original amount increases
Percent decrease – how much original amount decreases

\[
\text{percent change} = \frac{\text{amount of change}}{\text{original amount}}
\]

What do students need to know?

- An understanding of percent
- Part and whole
- Increase
- Decrease
- Original number
- Difference between percentage of and percent of change
Do they understand increase vs. decrease?

- If you buy a brand new car for $15,999, drive it off the lot, and get into an accident, the car will be worth $11,499. Does the car's value increase or decrease?
- The temperature at sunrise is 71 degrees Fahrenheit. At noon, the temperature is 84 degrees Fahrenheit. At sunset, it is 69 degrees Fahrenheit. Has the temperature had an increase or decrease from sunrise to sunset?
- A scuba diver jumps off a dive boat into the water and descends 30 feet below sea level. He rises 10 feet to swim above a coral head, then swims back down 8 feet to the top of a submerged wreck. Has his depth shown an increase or decrease from his initial descent?

Percent of Increase

- Tips
- Sales Tax
- Increase in Population

To calculate percent of increase

\[
\text{percent change} = \frac{\text{amount of increase}}{\text{original amount}}
\]
Calculating a Percent of Increase

In 1981, there were 25 endangered and threatened species of reptiles in the U. S. In 2015, there were 45 species. By what percent did the number of these reptile species change from 1981 to 2016?

\[
\frac{\text{amount of increase}}{\text{original amount}} = \frac{20}{25} = 80\%
\]

Is the amount of change an increase or a decrease? (increase)
What is the amount of change from 1981 to 2015? (45 - 25 = 20)
What is the original amount? (25)
Divide the amount of change by the original amount (20/25 = .8)
Write the quotient as a percent (.8 = 80% increase)

Calculating a Percent of Increase

198,000 people attended a concert in 2007. The number of attendees increase by 12% from 2007 to 2017. How many attendees attended in 2017?

\[
\text{Attendees in 2017} = \text{Attendees in 2007} + \text{Amount of increase}
\]

\[
= 198,000 + 198,000 \times 12\% \times (\text{Substitute})
\]

\[
= 198,000 + 198,000 \times 0.12 \text{ (write percent as a decimal)}
\]

\[
= 198,000 +
\]

\[
= 221,760 \text{ (Evaluate)}
\]
Percent of Decrease

- Discounts
- Sales
- Reduction in Population

To calculate percent of decrease

\[
\text{percent change} = \frac{\text{amount of decrease}}{\text{original amount}}
\]

Calculating a Percent of Decrease

A stock was worth $18.00 a share in 2000. In 2016, the same stock was worth $7.60 a share. What was the percent of change?

Is the amount of change an increase or a decrease? (decrease)

What is the amount of change from 2000 to 2016? ($10.40)

What is the original amount? ($18.00)

Divide the amount of change by the original amount (10.40/18)

Write the quotient as a percent (.57777... = 58% decrease)
Laptop Computer

You can now buy a laptop for 14% of the cost of a 1981 laptop. What was the cost in 1981?

<table>
<thead>
<tr>
<th>Laptop Price, 1981</th>
<th>14% of the original price</th>
</tr>
</thead>
<tbody>
<tr>
<td>$249</td>
<td>$430</td>
</tr>
<tr>
<td>$1,779</td>
<td></td>
</tr>
<tr>
<td>$3,071</td>
<td></td>
</tr>
</tbody>
</table>

Now and Then – NCTM: https://illuminations.nctm.org/Lesson.aspx?id=3121

Common Errors

• Using the wrong base when calculating change
• Not being able to differentiate between a quantity change and a percentage change
• Incorrectly changing a decimal to a percent
• Confusing "percentage of" situations with percent increase/decrease situations
• Not reading the situation (word problem) carefully
Resources

Art of Problem Solving: Percent Increase and Decrease Part 1
https://www.youtube.com/watch?v=vTPQV_M6ifI

Art of Problem Solving: Percent Increase and Decrease Part 2
https://www.youtube.com/watch?v=TbUlWJ9Ohw

How to Find the Percent Change Increase: The Easy Way
https://www.youtube.com/watch?v=YWOeN7hDD3E

How To Find Percent Change Decrease: The Easy Way
https://www.youtube.com/watch?v=fwhZ8TtIReY

Exponents and Roots

Oh My!
A Continuing Problem

Students think an exponent is the same as multiplication.

Multiplication = Repeated Addition

\[ 6 \times 3 = 18 \]
\[ 6 + 6 + 6 = 18 \]

Exponents = Repeated Multiplication

\[ 6^3 = 18 \]
\[ 6 \times 6 \times 6 = 216 \]

Do your students know the vocabulary?

Exponents

3 \[ \leftarrow \text{Exponent (or power)} \]

Base

Roots

\[ \text{Root index} \rightarrow 3 \]

Radical sign \[ \sqrt{27} \]

Radicand
Rules of Exponents Made Easier

The Math Dude – Law of Exponents - https://www.youtube.com/watch?v=g4bKGsC2IoY

Rules of Exponents

<table>
<thead>
<tr>
<th>Rule</th>
<th>Exemple</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x^1 = x$</td>
</tr>
<tr>
<td>2</td>
<td>$x^0 = 1$</td>
</tr>
<tr>
<td>3</td>
<td>$x^{-1} = \frac{1}{x^1}$</td>
</tr>
<tr>
<td>4</td>
<td>$(x^m)(x^n) = x^{m+n}$</td>
</tr>
<tr>
<td>5</td>
<td>$\frac{x^m}{x^n} = x^{m-n}$</td>
</tr>
<tr>
<td>6</td>
<td>$(x^m)^n = x^{m(n)}$</td>
</tr>
<tr>
<td>7</td>
<td>$(xy)^n = x^ny^n$</td>
</tr>
<tr>
<td>8</td>
<td>$(\frac{x}{y})^n = \frac{x^n}{y^n}$</td>
</tr>
<tr>
<td>9</td>
<td>$x^{-n} = \frac{1}{x^n}$</td>
</tr>
</tbody>
</table>
Squares and Square Roots of Positive Rational Numbers

Recommendations for Test-Takers

• Memorize the first 12 perfect squares (1, 4, 9, …, 144)
• Understand inverse relationships between pairs of squares and square roots (\(12^2 = \sqrt{144}\) and \(\sqrt{144} = 12\))
• Understand difference in squaring a negative number and the negative of a square number, i.e., \((-3)^2 = 9\) and \(-(-3)^2 = -9\)
• Practice computing with square and square roots that include fractions and decimals

Simplifying Radical Expressions

• \(\sqrt{9}\)
• Find the prime factors = \(\sqrt{3 \cdot 3}\)
• Bring any pairs outside the radical = 3

\[
\frac{\sqrt{9xy^2}}{\sqrt{3 \cdot 3 \cdot x \cdot y \cdot y}} = \frac{3y\sqrt{x}}{3y\sqrt{x}}
\]
Why do students need to know this?

An expression is shown:
\[ \sqrt{3} \cdot \sqrt[3]{5} \]

Simplify the expression completely. Leave your answer in radical form.

(NOTE: Click the symbol selector when you need to enter the radical sign.)

Last, but not least

The incredible zero
The Incredible Zero

- It is unique in representing nothingness.
- As a placeholder it gives our number system its power.
- It acquires different meaning based on its location. Think 30 versus 3,000.

The Origin of the Number Zero

http://www.smithsonianmag.com/history/origin-number-zero-180953392/#qagAYijydW3RXhhk.99

Properties of Zero

<table>
<thead>
<tr>
<th>Property</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>a + 0 = a</td>
<td>4 + 0 = 4</td>
</tr>
<tr>
<td>a − 0 = a</td>
<td>4 − 0 = 4</td>
</tr>
<tr>
<td>a × 0 = 0</td>
<td>6 × 0 = 0</td>
</tr>
<tr>
<td>0 / a = 0</td>
<td>0/3 = 0</td>
</tr>
<tr>
<td>a / 0 = undefined (dividing by zero is undefined)</td>
<td>7/0 = undefined</td>
</tr>
<tr>
<td>0^a = 0 (a is positive)</td>
<td>0^4 = 0</td>
</tr>
</tbody>
</table>

http://www.mathsisfun.com/numbers/zero.html
The Problem with Zero

\[ \frac{a}{0} \quad \frac{7}{0} \]

How many times can you throw nothing into no baskets?

You can express a fraction with 0 in the denominator, but it has no meaning.

Division by zero is undefined. Mathematicians have never defined the meaning because there is no good definition.

As many times as you want. It’s just not a real number.

To learn more: https://www.khanacademy.org/math/algebra/introduction-to-algebra/division-by-zero/v/why-dividing-by-zero-is-undefined

Calculator-Prohibited Items
Sample Items

Order Fractions and Decimals  Place the following numbers in order from greatest to least: 0.2, -1/2, 0.6, 1/3, 1, 0, 1/6

Factors and Multiples  Find the LCM that is necessary to perform the indicated operation. 7/6 – 1/4 =

Rules of Exponents  Simplify the following: (x^3)^5

Distance on a Number Line  Find the distance between two points -9 and -3 on a number line
Sample Items

Operations on Rational Numbers

Solve: \(3 \left( \frac{1}{2} \right) + 3 \frac{1}{2} = \)

Squares and Square Roots of Positive Rational Numbers

Find \(\sqrt{9}\)  Find \(\sqrt{24}\)

Cubes and Cube Roots of Rational Numbers

Find \((-4)^3\)

Undefined Value Over the Set of Real Numbers

Solve \((2x - 3) (x + 2) = 0\)

Tips for Building Foundational Skills

- Help students build their number sense
- Include opportunities for students to work together
- Provide plenty of practice with real-life situations included
- Set high expectations
https://ged.com

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Q & A
Thank you!

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