Math Grab Bag

A Workshop by GED Testing Service®

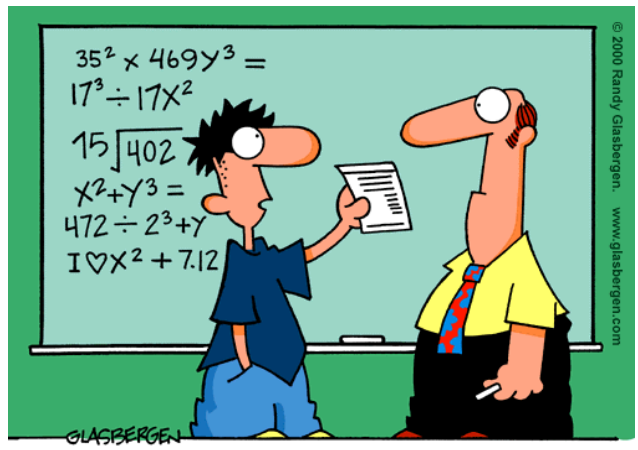
2

Session Objectives

- Discuss the challenges students have with key concepts in math
- Focus on the foundational "must haves" in mathematical reasoning
- Review tips and strategies for helping students build consistency in math
- Share resources

3

It Really Isn't Genetic



"I HAD MY DOCTOR DO A D.N.A. BLOOD ANALYSIS. AS I SUSPECTED, I'M MISSING THE MATH GENE."

4

Math involves…

- Memory
- Language
- Sequencing
- Spatial ordering
- Critical thinking
- Good problem-solving strategies
- Number sense
- Reasoning
- Making connections

5

In the Classroom, We Often…

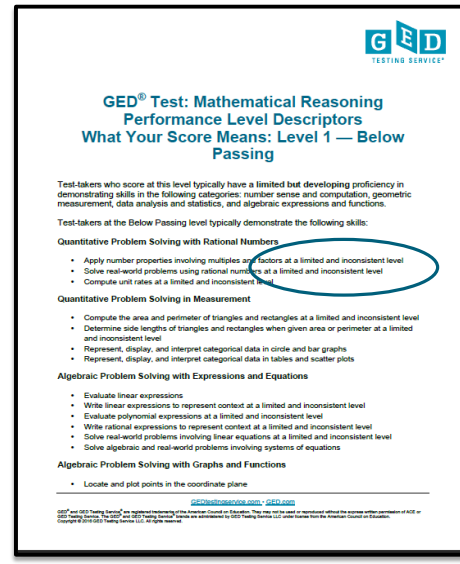
- Make assumptions about the presence or absence of foundational skills
- Introduce new concepts too rapidly
- Insufficiently support explanations and activities
- Provide insufficient practice
- Focus on facts versus concepts
- Limit access to manipulatives
- Limit connection of skills to real-life situations

6

What's the Problem?

Students at Level 1

- Have limited, but developing proficiency
- Perform some math skills at a basic level, such as
  - Putting fractions/decimals on a number line
  - Solving two-step arithmetic problems
- Are inconsistent in the application of skills
- Lack fluency in basic operations and mathematical properties



GED® Test: Mathematical Reasoning
Performance Level Descriptors
What Your Score Means: Level 1 — Below Passing

Test-takers who score at this level typically have a **limited but developing** proficiency in demonstrating skills in the following categories: number sense and computation, geometric measurement, data analysis and statistics, and algebraic expressions and functions.

Test-takers at the Below Passing level typically demonstrate the following skills:

Quantitative Problem Solving with Rational Numbers

- Apply number properties involving multiples and factors at a limited and inconsistent level
- Solve real-world problems using rational numbers at a limited and inconsistent level
- Compute unit rates at a limited and inconsistent level

Quantitative Problem Solving in Measurement

- Compute the area and perimeter of triangles and rectangles at a limited and inconsistent level
- Determine side lengths of triangles and rectangles when given area or perimeter at a limited and inconsistent level
- Represent, display, and interpret categorical data in circle and bar graphs
- Represent, display, and interpret categorical data in tables and scatter plots

Algebraic Problem Solving with Expressions and Equations

- Evaluate linear expressions
- Write linear expressions to represent context at a limited and inconsistent level
- Evaluate polynomial expressions at a limited and inconsistent level
- Write rational expressions to represent context at a limited and inconsistent level
- Solve real-world problems involving linear equations at a limited and inconsistent level
- Solve algebraic and real-world problems involving systems of equations

Algebraic Problem Solving with Graphs and Functions

- Locate and plot points in the coordinate plane

GEDTestingService.com • GED.com

7

Performance Level Descriptors (PLDs)

- Helpful tool for the classroom
- Explain in detail the skills students need to demonstrate to pass the test

8

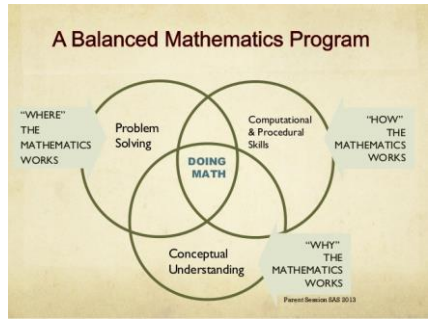
How to Use PLDs in the Classroom

Use PLDs to:

- **Tip 1:** Assess student's current skill level
- **Tip 2:** Determine when students are ready to test
- **Tip 3:** Shape learning activities
- **Tip 4:** Add perspective to lesson plans

9

Our Students Need…



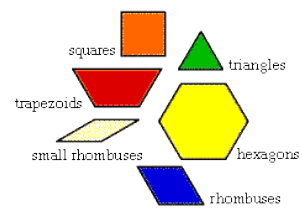
10

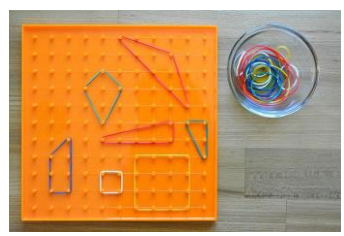
Understanding Skills Students Have

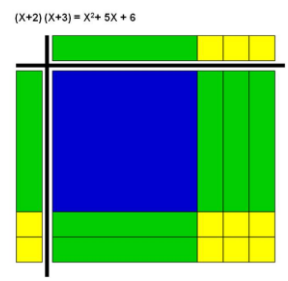
Low Intermediate Basic Education (4-5.9 GLE)	High Intermediate Basic Education (6-8.9 GLE)	Low Adult Secondary Education (9-10.9 GLE)
Students can perform with high accuracy all four basic math operations using whole numbers up to three digits and can identify and use all basic mathematical symbols.	Students can perform all four basic math operations with whole numbers and fractions; can determine correct math operations for solving narrative math problems and can convert fractions to decimals and decimals to fractions; and can perform basic operations on fractions.	Students can perform all basic math functions with whole numbers, decimals, and fractions; can interpret and solve simple algebraic equations, tables, and graphs and can develop own tables and graphs; and can use math in business transactions.

11

Best Practice for Building Skills

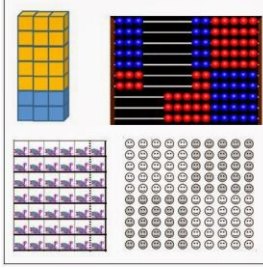
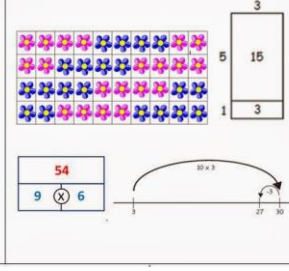






12

C-R-A – Essential for Understanding

Concrete	Representational	Abstract
Students manipulate hands-on, concrete materials	Students draw and observe diagrams, or watch the teacher touching and moving hands-on materials	Students use numbers and mathematical symbols
		x 4 Patterns 4 8 12 16 20 24 28 32 36 40 8 x 5 (4 x 2) x 5 4 x (2 x 5) 4 x 10 40 45 ÷ 5 (50-5) ÷ 5 (50÷5) - (5÷5) 10-1 9

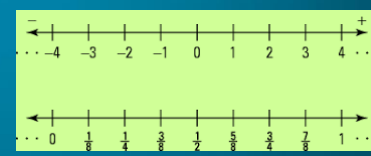
13

What are the Foundational "Must-Haves" in Mathematical Reasoning

14

Building Number Sense

Use a Number Line



15

Students with Number Sense…

- Think and reason flexibly with numbers
- Use numbers to solve problems
- Spot unreasonable answers
- Understand how to put numbers together and take them apart
- Understand number relationships

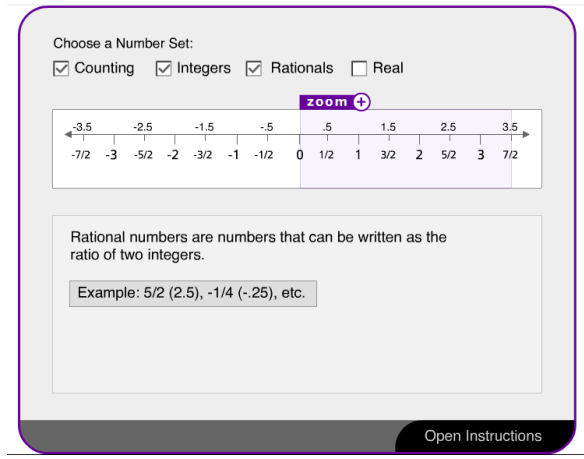
16

But they are just numbers…

Type of Number	Quick Description
Counting Numbers	{1, 2, 3, …}
Whole Numbers	{0, 1, 2, 3, …}
Integers	{…, -3, -2, -1, 0, 1, 2, 3, …}
Rational Numbers	p/q – p and q are integers, q is not zero
Irrational Numbers	π – 3.14159265358979323856… cannot be written as a simple fraction $\sqrt{3}$, $\sqrt{99}$
Real Numbers	Rational and Irrational

17

Defining Numbers

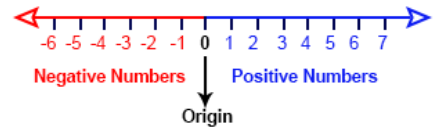


https://unctv.pbslearningmedia.org/resource/mgbh.math.ns.numbline/building-a-number-line/#.WU1B-IWcHnM

18

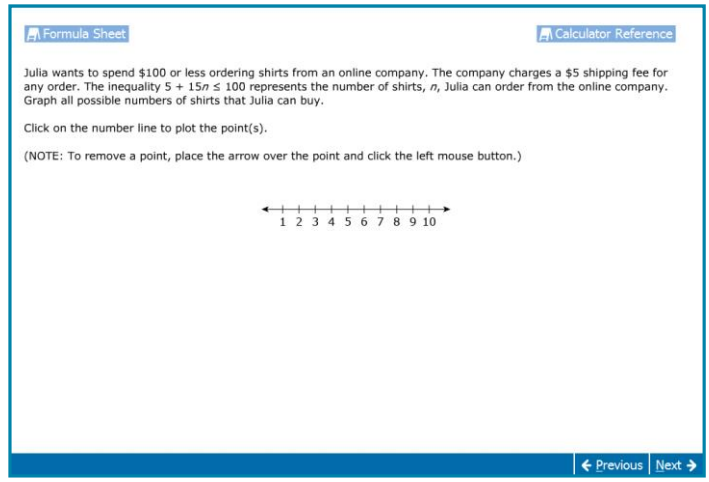
The Number Line

- Provides a model for basic operations for all rational numbers
- Is a spatial object
- Allows students to situate themselves spatially in mathematics
- Permits students to conceptualize mathematics



19

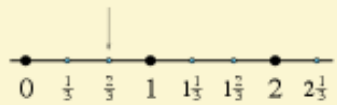
Just a Sample



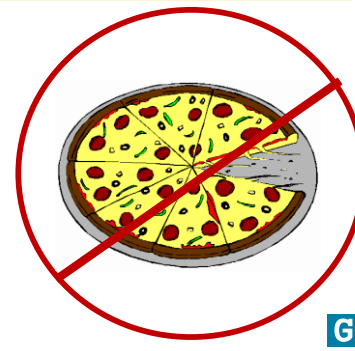
20

Fractions

Students need to learn to *locate* a fraction on a number line.

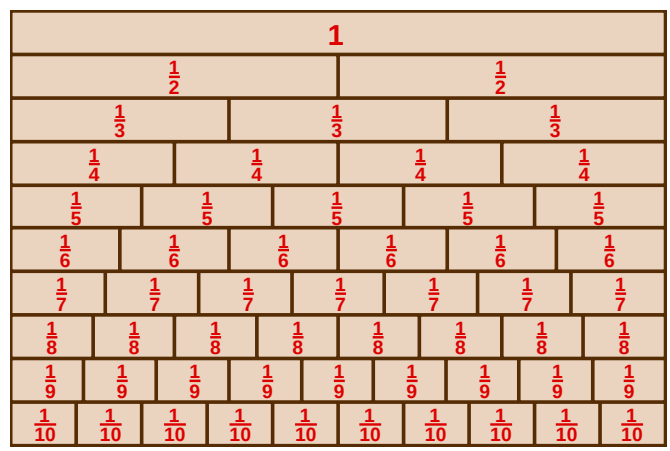


Use the number line so students understand that fractions are numbers and not just part of a pizza.



21

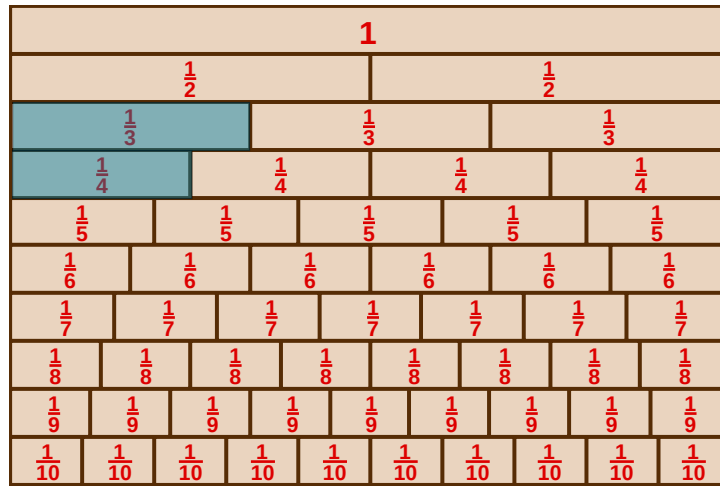
Fraction Tiles



A linear model gives an overview and shows relationships.

22

Fraction Tiles



What is more, 1/4 or 1/3? What is more, 1/9 or 1/10?

23

Can students identify the problem?

a) $\frac{2}{5}+\frac{1}{3}=\frac{3}{8}$

b) $\frac{2}{7}+\frac{1}{3}=\frac{2}{21}+\frac{1}{21}=\frac{3}{21}$

c) $\frac{3}{8}+\frac{1}{4}=\frac{9}{32}+\frac{8}{32}=\frac{17}{32}$

d) $\frac{8}{9}+\frac{1}{4}=\frac{32}{36}+\frac{9}{36}=\frac{41}{72}$

e) $\frac{7}{12}+\frac{5}{6}=\frac{14}{24}+\frac{20}{24}=\frac{34}{24}=\frac{17}{12}$

f) $6\frac{1}{2}-4\frac{2}{3}=6-4+\frac{1}{2}-\frac{2}{3}=2+\frac{3}{6}-\frac{4}{6}=2\frac{1}{6}$

g) $5\frac{2}{5}+1\frac{3}{4}=5\frac{8}{20}+1\frac{15}{20}=6\frac{23}{20}=7\frac{3}{20}$

h) $\frac{1}{2}\times\frac{1}{5}=\frac{5}{10}\times\frac{2}{10}=\frac{10}{100}=\frac{1}{10}$

i) $2\frac{1}{2}\div1\frac{3}{4}=\frac{5}{2}\times\frac{7}{4}=\frac{35}{8}=4\frac{3}{8}$

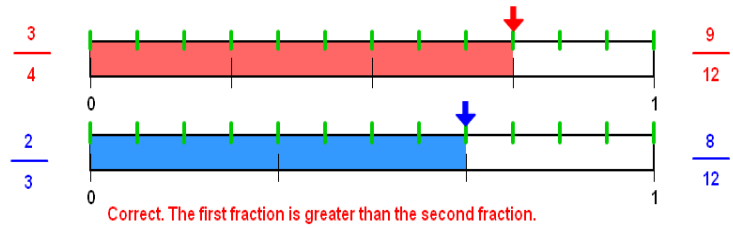
j) $\frac{17}{20}-\frac{4}{5}\times\frac{3}{8}=\frac{17}{20}-\frac{16}{20}\times\frac{3}{8}=\frac{1}{20}\times\frac{3}{8}=\frac{4}{160}=\frac{1}{40}$

"Five out of four people have trouble with fractions."
-- Steven Wright

24

Can Students Use a Number Line?

The fractions 3/4 and 2/3 are pictured with number lines below:



Correct. The first fraction is greater than the second fraction.

$\frac{3}{4} > \frac{2}{3}$

25

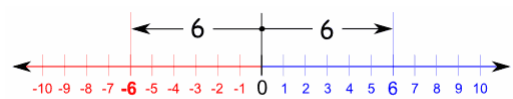
Absolute Value

Absolute Value means how far a number is from 0.

- Remove any negative sign and think of all numbers as positive
- Recognize symbol used to represent absolute value

$|-5| = 5$

$|7| = 7$



"6" is 6 away from zero,
and "−6" is **also** 6 away from zero.

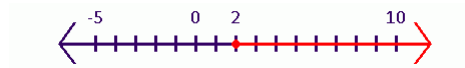
So the absolute value of 6 is **6**,
and the absolute value of −6 is also **6**

26

Inequalities

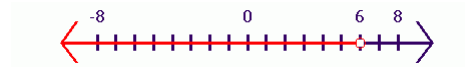
An inequality is a math statement that defines a range of values.

Jeffrey runs at least two miles every day.



On November 28, the temperature in North Pole, Alaska is expected to be greater than -4° and less than 9°

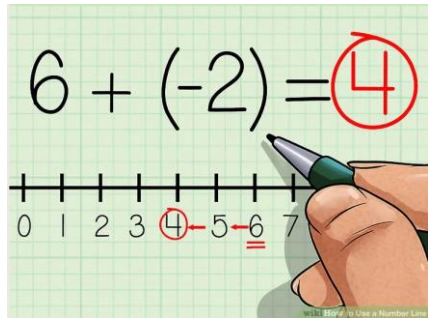
T < 6

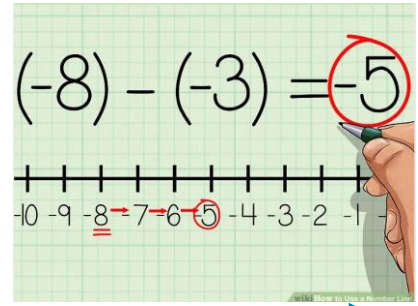


https://www.gedtestingservice.com/educators/exploring-the-2014-ged-test-webinar-archive

27

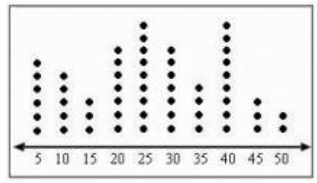
Operations with Positive and Negative Integers

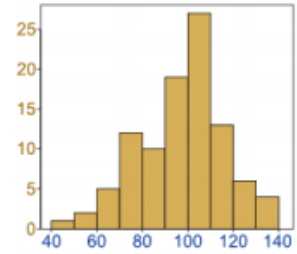


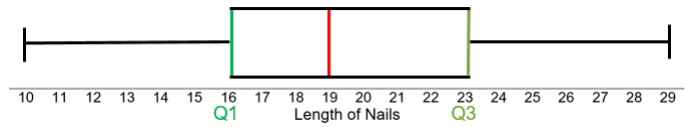


28

Data Displays



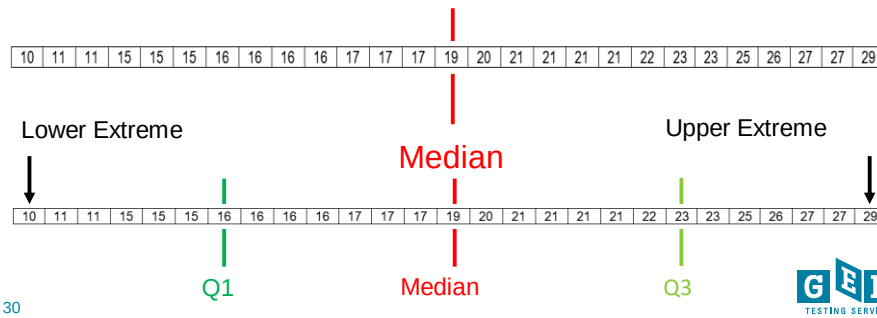




29

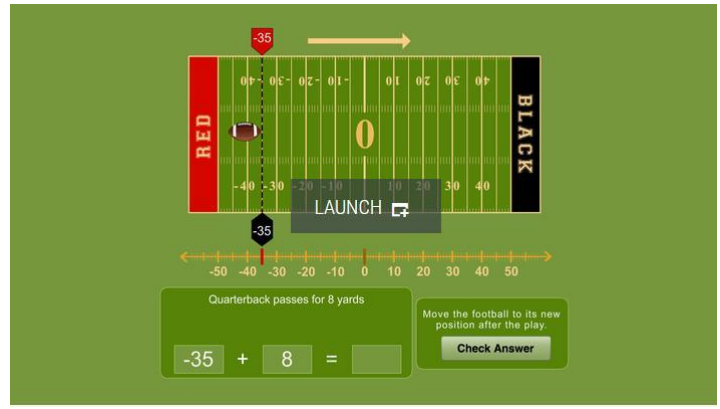
Essential Skills for Box Plots

Arrange data in order
Find median for all data
Find median for lower quartile
Find median for upper quartile
Find the range of the extreme values



30

Number Lines and the NFL



https://unctv.pbslearningmedia.org/resource/mket-math-ns-ratnumb/football/#.WU0tDGgrKUk

31

Resources

Helping with Math - Number Line Generator

http://www.helpingwithmath.com/printables/others/NumberLineGenerator01.htm

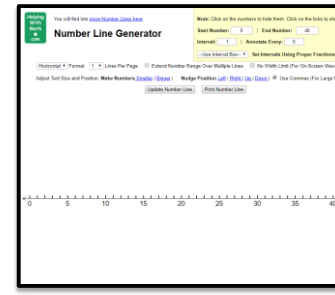
Math Warehouse – Number Line Graph Maker

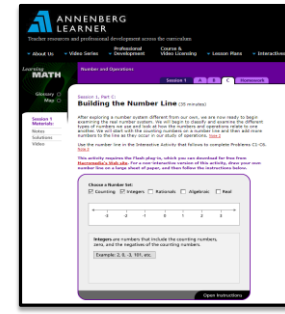
http://www.mathwarehouse.com/number-lines/number-line-maker.php

Math is Fun – Number Lines (Inequalities, Operations, etc.)

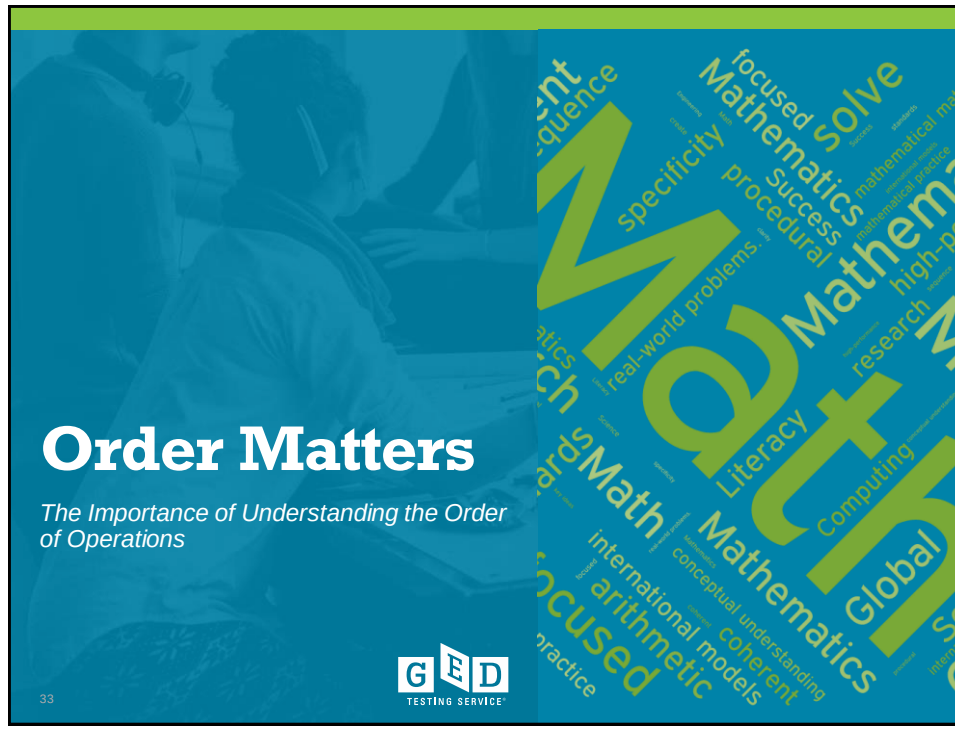
http://www.mathsisfun.com/number-line.html

Annenberg Learner – Building the Number Line
http://www.learner.org/courses/learningmath/number/session1/part_c/index.html





32



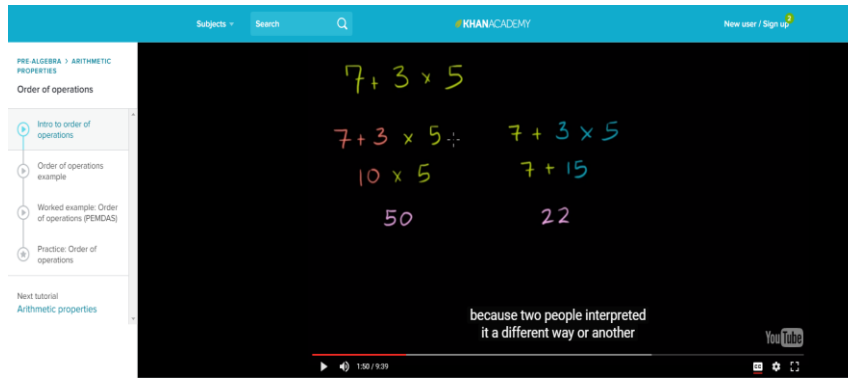
Why Bother?

Here is your problem: 4 + 2×3 =
Is the answer 18 or 10?

- Avoid confusion in how problems are solved
- Set up rules of precedence or rank of operations
- Is critical to simplifying and solving different algebra problems

34

Answer the Why



https://www.khanacademy.org/math/pre-algebra/pre-algebra-arith-prop/pre-algebra-order-of-operations/v/introduction-to-order-of-operations

35

Get Rid of Misconceptions about Order of Operations

Misconception 1 - All multiplication should happen before division.

Incorrect	Correct
$12 \div 3 \times 4$	$12 \div 3 \times 4$
$12 \div 12$	4×4
1	16

Misconception 2 – All addition comes before subtraction.

Incorrect	Correct
$4+10-5+8$	$4+10-5+8$
$14-13$	$14-5+8$
1	$9+8$
	17

GROUPINGS ( ) { } [ ]
EXPONENTS N^2
MULTIPLY/**D**IVIDE ÷/×
(LEFT TO RIGHT)
SUBTRACT/**A**DD +/–
(LEFT TO RIGHT)

Parenthesis
Exponents
Multiply / **D**ivide
Add + **S**ubtract

Essential Understanding

1. **Parentheses and Brackets** from the inside out
2. **Exponents** of numbers or parentheses
3. **Multiplication and Division** in the order they appear
4. **Addition and Subtraction** in the order they appear

37

Your Turn

What is the value of $6 \div 3 + 4 \times 2$?

So we do the division and multiplication before any addition or subtraction:

$$6 \div 3 + 4 \times 2$$
$$= 2 + 4 \times 2$$
$$= 2 + 8$$
$$= 10$$

38

Your Turn

What is the value of $6 \times 4 - 12 \div 3 - 8$?

We do 6×4 and $12 \div 3$ first, then the subtractions:

$$\begin{aligned} & 6 \times 4 - 12 \div 3 - 8 \\ = \; & 24 - 12 \div 3 - 8 \\ = \; & 24 - 4 - 8 \\ = \; & 20 - 8 \\ = \; & 12 \end{aligned}$$

39

Your Turn

What is the value of $20 - (3 \times 2^3 - 5)$?

We start inside the **P**arentheses, and then use "**E**xponents" first:

$20 - (3 \times 2^3 - 5) = 20 - (3 \times 8 - 5)$

[Because 2^3 means $2 \times 2 \times 2 = 8$, **not** $2 \times 3 = 6$]

Next **M**ultiply:
$20 - (3 \times 8 - 5) = 20 - (24 - 5)$

Next **S**ubtract (still working inside the parentheses):
$20 - (24 - 5) = 20 - 19$

Now the **P**arentheses are completed, the last operation is **S**ubtract:
$20 - 19 = 1$

40

Your Turn

What is the value of $(12 \div 3 + 4) - (4^2 - 6 \times 2)$?

$$(12/3+4)-(4^2-6\times 2)$$
$$= (4+4)-(4^2-6\times 2)$$
$$= 8-(4^2-6\times 2)$$
$$= 8-(16-6\times 2)$$
$$= 8-(16-12)$$
$$= 8-4$$
$$= 4$$

41

What's Your Sign?

In the equation below, replace each question mark with one of the four mathematical signs: +,-, ×, or ÷. Each sign can be used only once. Fill in the blanks to solve the equation. (Hint: the first sign is +.)

7 ? 5 ? 4 ? 7 ? 6 = 15

What is the first step that you need to take in order to solve the equation? Write your answer in the question box.

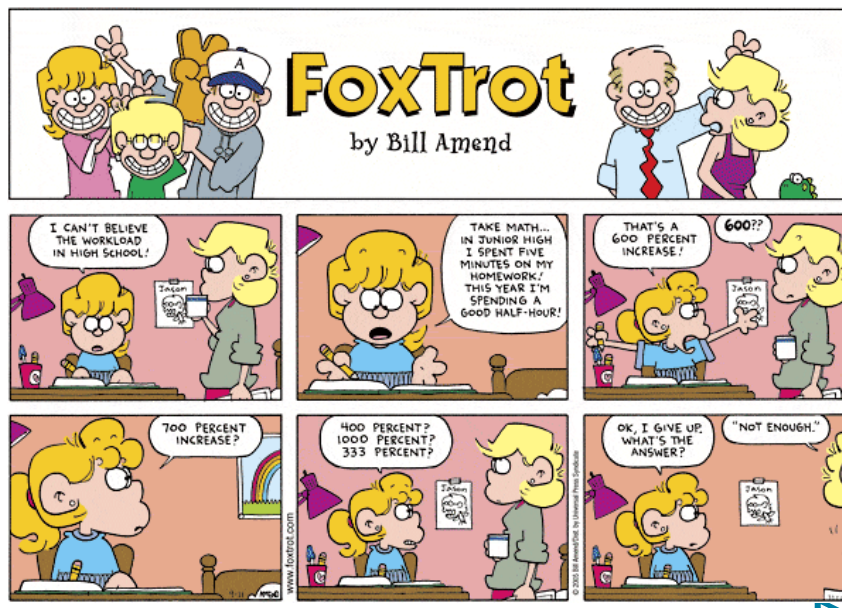
$(7 + 5) \div 4 \times 7 - 6 = 15$

Percent Change

From Shopping to Identifying Trends

43

44



Do your students know the vocabulary?

Ratio – a comparison between two different values

Percent of change – ratio of the amount of change to the original amount

Percent increase – how much original amount increases

Percent decrease – how much original amount decreases

$$\text{percent change} = \frac{\text{amount of change}}{\text{original amount}}$$

45

What do students need to know?

- An understanding of percent
- Part and whole
- Increase
- Decrease
- Original number
- Difference between percentage of and percent of change

46

Do they understand increase vs. decrease?

- If you buy a brand new car for $15,999, drive it off the lot, and get into an accident, the car will be worth $11,499. Does the car's value increase or decrease?
- The temperature at sunrise is 71 degrees Fahrenheit . At noon, the temperature is 84 degrees Fahrenheit. At sunset, it is 69 degrees Fahrenheit. Has the temperature had an increase or decrease from sunrise to sunset?
- A scuba diver jumps off a dive boat into the water and descends 30 feet below sea level. He rises 10 feet to swim above a coral head, then swims back down 8 feet to the top of a submerged wreck. Has his depth shown an increase or decrease from his initial descent?

47

Percent of Increase

- Tips
- Sales Tax
- Increase in Population

To calculate percent of increase

$$\text{percent change} = \frac{\text{amount of increase}}{\text{original amount}}$$

48

Calculating a Percent of Increase

In 1981, there were 25 endangered and threatened species of reptiles in the U. S. In 2015, there were 45 species. By what percent did the number of these reptile species change from 1981 to 2016?

80%

$$\text{percent change} = \frac{\text{amount of increase}}{\text{original amount}} = \frac{20}{25}$$

Is the amount of change an increase or a decrease? (increase)
What is the amount of change from 1981 to 2015? (45 - 25 = 20)
What is the original amount? (25)
Divide the amount of change by the original amount (20/25 = .8)
Write the quotient as a percent (.8 = 80% increase)

49

Calculating a Percent of Increase

198,000 people attended a concert in 2007. The number of attendees increase by 12% from 2007 to 2017. How many attendees attended in 2017?

Attendees in 2017 = **Attendees in 2007** + **Amount of increase**

= 198,000 + 198,000 x 12% x (Substitute)
= 198,000 + 198,000 x 0.12 (write percent as a decimal)
=198,000 +
= 221,760 (Evaluate)

50

Percent of Decrease

- Discounts
- Sales
- Reduction in Population

To calculate percent of decrease

$$\text{percent change} = \frac{\text{amount of decrease}}{\text{original amount}}$$

51

Calculating a Percent of Decrease

A stock was worth $18.00 a share in 2000. In 2016, the same stock was worth $7.60 a share. What was the percent of change?

$$\frac{10.40}{18.00} = 58\%$$

Is the amount of change an increase or a decrease? (decrease)

What is the amount of change from 2000 to 2016? ($10.40)

What is the original amount? ($18.00)

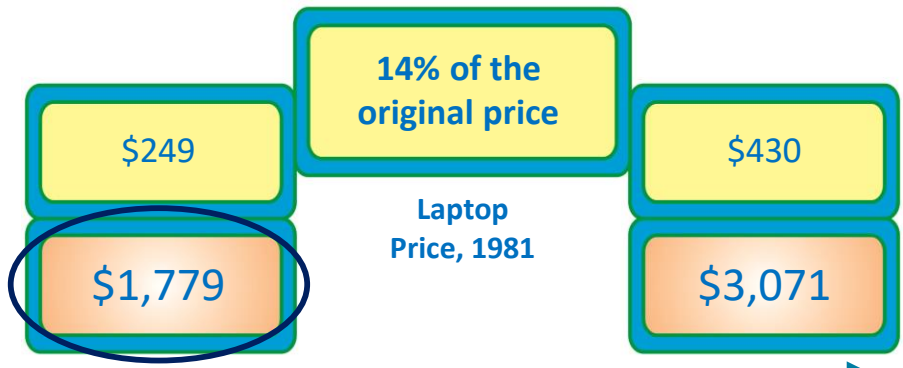
Divide the amount of change by the original amount (10.40/18)

Write the quotient as a percent (.57777 . . . = 58% decrease)

52

Laptop Computer

You can now buy a laptop for 14% of the cost of a 1981 laptop. What was the cost in 1981?



Now and Then – NCTM: https://illuminations.nctm.org/Lesson.aspx?id=3121

Common Errors

- Using the wrong base when calculating change
- Not being able to differentiate between a quantity change and a percentage change
- Incorrectly changing a decimal to a percent
- Confusing "percentage **of**" situations with percent increase/decrease situations
- Not reading the situation (word problem) carefully

54

Resources

Art of Problem Solving: Percent Increase and Decrease Part 1

https://www.youtube.com/watch?v=vTPQV_M6tfI

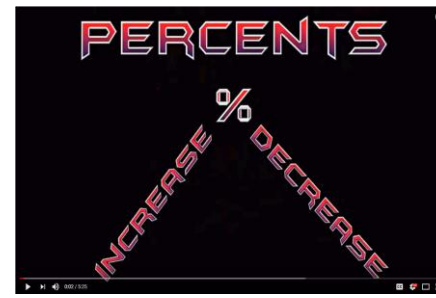
Art of Problem Solving: Percent Increase and Decrease Part 2

https://www.youtube.com/watch?v=TbUlfWJ9Ohw

How to Find the Percent Change Increase: The Easy Way https://www.youtube.com/watch?v=YWOeN7hDD3E

How To Find Percent Change Decrease: The Easy Way

https://www.youtube.com/watch?v=fwhZ8ITiReY



55

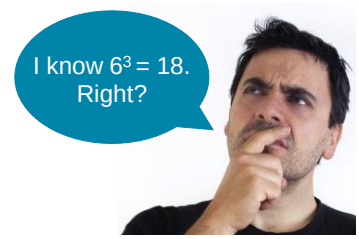
Exponents and Roots

Oh My!

56

A Continuing Problem

Students think an exponent is the same as multiplication.



Multiplication = Repeated Addition

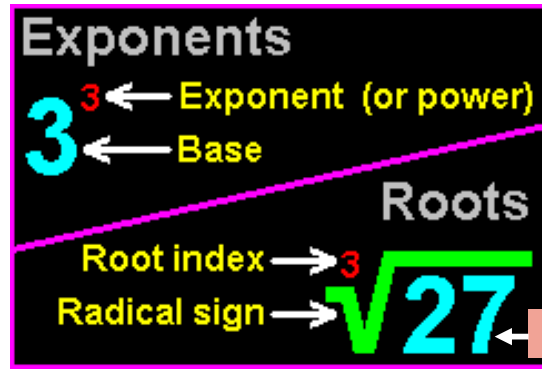
$6 \times 3 = 18$
$6 + 6 + 6 = 18$

Exponents = Repeated Multiplication

$6^3 = 18$
$6 \times 6 \times 6 = 216$

57

Do your students know the vocabulary?



58

Rules of Exponents Made Easier

The Math Dude – Law of Exponents - https://www.youtube.com/watch?v=g4bKGsC2IoY

59

Rules of Exponents

	Rule	Example
1	$x^1 = x$	$5^1 = 5$
2	$x^0 = 1$	$5^0 = 1$
3	$x^{-1} = \frac{1}{x^1}$	$5^{-1} = \frac{1}{5}$
4	$(x^m)(x^n) = x^{m+n}$	$(x^2)(x^3) = x^{2+3} = x^5$
5	$\frac{x^m}{x^n} = x^{m-n}$	$\frac{x^3}{x^2} = x^{3-2} = x^1$
6	$(x^m)^n = x^{(m)(n)}$	$(x^3)^2 = x^{(3)(2)} = x^6$
7	$(xy)^n = x^n y^n$	$(xy)^3 = x^3 y^3$
8	$(\frac{x}{y})^n = \frac{x^n}{y^n}$	$(\frac{x}{y})^3 = \frac{x^3}{y^3}$
9	$x^{-n} = \frac{1}{x^n}$	$x^{-2} = \frac{1}{x^2}$

60

Squares and Square Roots of Positive Rational Numbers

Recommendations for Test-Takers

- Memorize the first 12 perfect squares (1, 4, 9, …, 144)
- Understand inverse relationships between pairs of squares and square roots ($12^2 = \sqrt{144}$ and $\sqrt{144} = 12$)
- Understand difference in squaring a negative number and the negative of a square number, i.e., $(-3)^2 = 9$ and $-(-3)^2 = -9$
- Practice computing with square and square roots that include fractions and decimals

61

Simplifying Radical Expressions

- $\sqrt{9}$
- Find the prime factors = $\sqrt{3 \cdot 3}$
- Bring any pairs outside the radical = 3

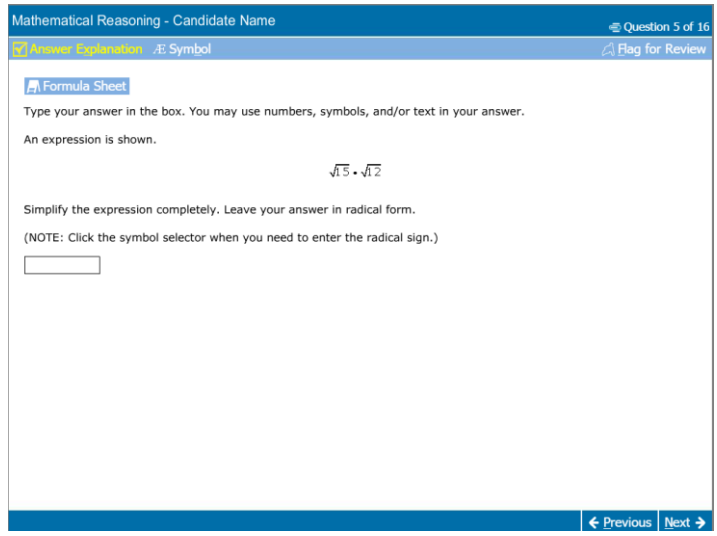
$\sqrt{9xy^2}$

$\sqrt{3 \cdot 3 \cdot x \cdot y \cdot y}$

$3y\sqrt{x}$

62

Why do students need to know this?



63

Last, but not least

The incredible zero

64

The Incredible Zero

- It is unique in representing nothingness.
- As a placeholder it gives our number system its power.
- It acquires different meaning based on its location. Think 30 versus 3,000.

The Origin of the Number Zero

http://www.smithsonianmag.com/history/origin-number-zero-180953392/#qagAYijydW3RXhhk.99

65

66

Properties of Zero

Property	Example
$a + 0 = a$	$4 + 0 = 4$
$a - 0 = a$	$4 - 0 = 4$
$a \times 0 = 0$	$6 \times 0 = 0$
$0 / a = 0$	$0/3 = 0$
$a / 0$ = undefined (dividing by zero is undefined)	$7/0$ = undefined
$0^a = 0$ (a is positive)	$0^4 = 0$

http://www.mathsisfun.com/numbers/zero.html

The Problem with Zero

$\frac{a}{0}$ $\frac{7}{0}$

You can express a fraction with 0 in the denominator, but it has no meaning.

Division by zero is undefined. Mathematicians have never defined the meaning because there is no good definition.

How many times can you throw nothing into no baskets?

As many times as you want. It's just not a real number.

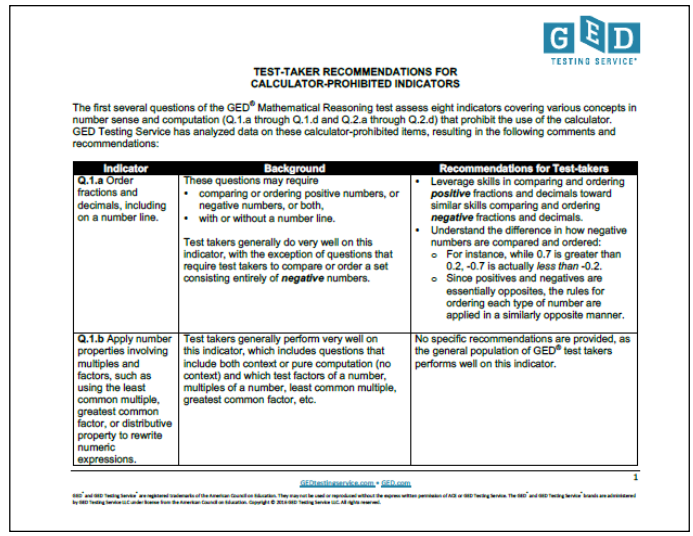
To learn more: https://www.khanacademy.org/math/algebra/introduction-to-algebra/division-by-zero/v/why-dividing-by-zero-is-undefined

67

Calculator-Prohibited Items

68

Calculator-Prohibited Indicators



TEST-TAKER RECOMMENDATIONS FOR CALCULATOR-PROHIBITED INDICATORS

The first several questions of the GED® Mathematical Reasoning test assess eight indicators covering various concepts in number sense and computation (Q.1.a through Q.1.d and Q.2.a through Q.2.d) that prohibit the use of the calculator. GED Testing Service has analyzed data on these calculator-prohibited items, resulting in the following comments and recommendations:

Indicator	Background	Recommendations for Test-takers
Q.1.a Order fractions and decimals, including on a number line.	These questions may require • comparing or ordering positive numbers, or negative numbers, or both, • with or without a number line. Test takers generally do very well on this indicator, with the exception of questions that require test takers to compare or order a set consisting entirely of ***negative*** numbers.	• Leverage skills in comparing and ordering ***positive*** fractions and decimals toward similar skills comparing and ordering ***negative*** fractions and decimals. • Understand the difference in how negative numbers are compared and ordered: o For instance, while 0.7 is greater than 0.2, -0.7 is actually *less than* -0.2. o Since positives and negatives are essentially opposites, the rules for ordering each type of number are applied in a similarly opposite manner.
Q.1.b Apply number properties involving multiples and factors, such as using the least common multiple, greatest common factor, or distributive property to rewrite numeric expressions.	Test takers generally perform very well on this indicator, which includes questions that include both context or pure computation (no context) and which test factors of a number, multiples of a number, least common multiple, greatest common factor, etc.	No specific recommendations are provided, as the general population of GED® test takers performs well on this indicator.

1

GEDtestingservice.com • GED.com

69

Sample Items

Order Fractions and Decimals	Place the following numbers in order from greatest to least: 0.2, -1/2, 0.6, 1/3, 1, 0, 1/6
Factors and Multiples	Find the LCM that is necessary to perform the indicated operation. 7/6 – 1/4 =
Rules of Exponents	Simplify the following: $(x^3)^5$
Distance on a Number Line	Find the distance between two points -9 and -3 on a number line

70

Sample Items

Operations on Rational Numbers	Solve: 3 (½) ÷ 3 ½ =
Squares and Square Roots of Positive Rational Numbers	Find $\sqrt{9}$ Find $\sqrt{24}$
Cubes and Cube Roots of Rational Numbers	Find $(-4)^3$
Undefined Value Over the Set of Real Numbers	Solve $(2x - 3)(x + 2) = 0$

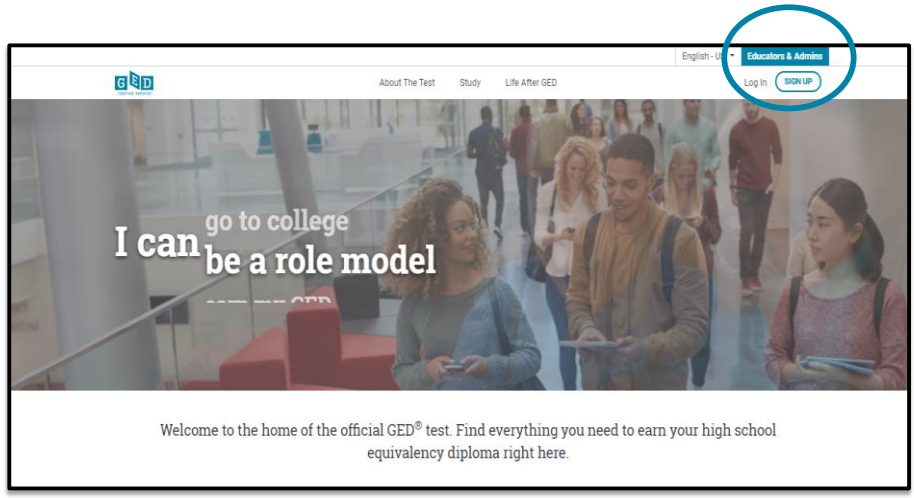
71

Tips for Building Foundational Skills

- Help students build their number sense
- Include opportunities for students to work together
- Provide plenty of practice with real-life situations included
- Set high expectations

72

https://ged.com



73

Q & A

74