yoűnspire

## Grasping GED ${ }^{\circledR}$ Higher Order Math Concepts for Deeper Understanding

July 18, 2023



## GRD <br> TESTING SERVICE ${ }^{\oplus}$

## Facilitator

## Ronald Cruz

- Adult Education Coordinator, Hillsborough County Public Schools, Tampa, FL
- Former Mathematics and Physics Teacher

- National Trainer, GED®
- Trainer and Content Developer, Florida IPDAE
- Statewide Trainer and Consultant for Delaware, Georgia, Maryland, Mississippi, and South Carolina


## In this session, we will

- Discuss how to push high achieving students to score at a college ready or college ready + credit level on the mathematical reasoning test.
- Focus on critical performance gaps based on field testing data
- Explore strategies, resources and ideas to address these performance gaps.



## Three Score Level Indicators on GED Ready ${ }^{\circledR}$

College Ready + Credit: 175-200

College Ready: 165-174


High School Equivalency: 145-164


## How do we make/high

 - performing GED students perform even better?TESTING SERVICE ${ }^{\circ}$

## Performance Gaps

1. Non-Calculator Items
2. Exponents and Roots
3. Three-Dimensional Shapes
4. Algebraic Computations
5. Inequalities
6. Slope and Graphing Linear Equations
7. Multiple Correct Answers

Tuesdays for Teachers:
GED Knowledge \& Skill Gaps - Math Session 1 - Oct. 26, 2021
Session 2 - Nov. 16, 2021

## CalculatorProhibited Indicators

Performance Gap 1


## Non-Calculator Items

1. Ordering Rational Numbers
2. Factors and Multiples
3. Distance on a Number Line
4. Operations on Rational Numbers

5. Rules of Exponents
6. Squares and Square Roots of Positive Rational Numbers
7. Cubes and Cube Roots of Rational Numbers
8. Undefined Value Over the Set of Real Numbers

## 60 Second Challenge: Ordering Rational Numbers



Arrange the given numbers in ascending order.

$$
\frac{5}{8},-3,0.8314, \frac{1}{16},-\pi, 0.4823, \frac{5}{12}
$$

## Let's Try Again


younspire GED

## Ordering Rational Numbers: Use a Number Line

$$
\frac{5}{8},-3,0.8314, \frac{1}{16},-\pi, 0.4823, \frac{5}{12}
$$

1. Draw a number line and mark benchmark numbers.
2. Plot the easiest given numbers in reference to benchmark numbers.
3. Compare and plot the last remaining numbers.


## Factors and Multiples

-Greatest Common Factor - Used to simplify fractions

$$
\text { Example: } \frac{22}{12}=\frac{22 \div 2}{12 \div 2}=\frac{11}{6}
$$

-Least Common Multiple (LCM) - Used to add/subtract fractions with unlike denominators.

$$
\text { Example: } \frac{7}{6}-\frac{1}{4}=\frac{14-3}{(12}=\frac{11}{12}
$$

## Greatest Common Factor

Example 1: Find the GCF of 35 and 42.

$\mathrm{GCF}=2 \times 3=6$
younspire 븡

## Least Common Multiple (LCM)

Example 1: Find the LCM of 8,4 , and 6 .

Method 1: Listing Multiples
$8 \rightarrow 8,16,24,32,40,48$
$4 \rightarrow 4,8,12,16,20,24,28,32$
$6 \rightarrow 6,12,18,24,30,36$

LCM = $\mathbf{2 4}$

Example 2: Find the LCM that is necessary to perform the indicated operation: $\frac{3}{8}-\frac{1}{42}=$


## Distance on a Number Line

The distance is how far apart points are on a number line.

## 5 units



Find the distance between -24 and 13 on a number line.

## Distance on a Number Line

The distance between two points $A$ and $B$ on a number line is:

$$
=|A-B|
$$

Example:
Find the distance between -24 and 13 on a number line.

$$
\begin{gathered}
=|-24-13| \\
=|-37| \\
=37
\end{gathered}
$$

Absolute Value is the distance of a number from zero on a number line.

## Operations on Rational Numbers

By MathTricks on Facebook Reels https://www.facebook.com/reel/657776426 087451/


Fractions | Adding and subtracting fractions|Dividing and multiply fractions \#fraction \#reels - aks - Original at

## finally found the square root!

## Exponents and Roots/Radicals

Performance Gap 2


## Rules of Exponents

Workbook P. 3

## Name

Product

$$
x^{3} \cdot x^{4}=x^{3+4}=x^{7}
$$

Quotient

$$
a^{m} \div a^{n}=a^{m-n}
$$

$$
p^{5} \div p^{2}=p^{5-2}=p^{3}
$$

Power of a Power

$$
\left(z^{3}\right)^{2}=z^{3 \cdot 2}=z^{6}
$$

Power of a Product

$$
(a b)^{m}=a^{m} b^{m}
$$

$$
\left(\frac{a}{b}\right)^{m}=\frac{a^{m}}{b^{m}}
$$

## Rule

$$
a^{m} \cdot a^{n}=a^{m+n}
$$

$$
\left(a^{m}\right)^{n}=a^{m n}
$$

$$
(3 y)^{2}=3^{2} y^{2}=9 y^{2}
$$

Power of a Quotient

$$
\left(\frac{5}{3}\right)^{2}=\frac{5^{2}}{3^{2}}=\frac{25}{9}
$$

Zero Exponent
Negative Exponent

$$
a^{-m}=\frac{1}{a^{m}}
$$

$$
b^{-3}=\frac{1}{b^{3}} ; 5^{-2}=\frac{1}{5^{2}}
$$

Fractional Exponent

$$
a^{\frac{m}{n}}=\sqrt[n]{a^{m}} \quad 4^{\frac{3}{2}}=\sqrt[2]{4^{3}}=\sqrt{64}=8
$$

## Square and Square Root Tricks (Part l)

Combining of Similar Radicals

$$
\begin{aligned}
& a \sqrt{b}+a \sqrt{b}=(a+a) \sqrt{b} \\
& a \sqrt{b}-c \sqrt{b}=(a-c) \sqrt{b}
\end{aligned}
$$

Example 1: $\quad 2 \sqrt{5}+6 \sqrt{5}=(2+6) \sqrt{5}=8 \sqrt{5}$

Example 2: $\quad 3 \sqrt{2}-5 \sqrt{2}=(3-5) \sqrt{2}=-2 \sqrt{2}$

## Square and Square Root Tricks (Part 2)

Splitting Products

$$
\begin{aligned}
& \sqrt{x^{3}}=\sqrt{x^{2} \cdot x}=\sqrt{x^{2}} \sqrt{x}=|x| \sqrt{x} \\
& \sqrt{20}=\sqrt{4 \cdot 5}=\sqrt{4} \cdot \sqrt{5}=|2| \sqrt{5}
\end{aligned}
$$

Splitting Quotients

$$
\sqrt{\frac{x^{2}}{y^{2}}}=\frac{\sqrt{x^{2}}}{\sqrt{y^{2}}}=\frac{x}{y} \quad \sqrt{\frac{4}{25}}=\frac{\sqrt{4}}{\sqrt{25}}=\frac{|2|}{|5|}
$$

## Square and Square Root Exercise

Simplify $2 \sqrt{2}(2 \sqrt{3}+3 \sqrt{3})$

$$
=2 \sqrt{2}(5 \sqrt{3})=10 \sqrt{2 \cdot 3}=10 \sqrt{6}
$$

Simplify $3 \sqrt{24 x^{3}}$

$$
=3 \sqrt{4 \cdot 6 \cdot x^{2} \cdot x}=3 \cdot 2 \cdot x \sqrt{6 x}=6 x \sqrt{6 x}
$$

Simplify $(-4 \sqrt{2})^{2}$

$$
=(-4)^{2}(\sqrt{2})^{2}=16 \cdot 2=32
$$

Simplify $\sqrt{\frac{12 x^{2}}{4}}=\frac{\sqrt{12 x^{2}}}{\sqrt{4}}=\frac{\sqrt{4 \cdot 3 \cdot x^{2}}}{2}=\frac{2 x \sqrt{3}}{2}=x \sqrt{3}$

## Cube and Cube Root Tricks (Part l)

Combining of Similar Radicals

$$
\begin{aligned}
& a \sqrt[3]{b}+a \sqrt[3]{b}=(a+a) \sqrt[3]{b} \\
& a \sqrt[3]{b}-c \sqrt[3]{b}=(a-c) \sqrt[3]{b}
\end{aligned}
$$

Example 1: $\quad 2 \sqrt[3]{5}+6 \sqrt[3]{5}=(2+6) \sqrt[3]{5}=8 \sqrt[3]{5}$

Example 2: $\quad 3 \sqrt[3]{2}-5 \sqrt[3]{2}=(3-5) \sqrt[3]{2}=-2 \sqrt[3]{2}$

## Cube and Cube Root Tricks (Part 2)

Splitting Products

$$
\begin{aligned}
& \sqrt[3]{x^{4}}=\sqrt[3]{x^{3} \cdot x}=\sqrt[3]{x^{3}} \cdot \sqrt[3]{x}=x \sqrt[3]{x} \\
& \sqrt[3]{16}=\sqrt[3]{8 \cdot 2}=\sqrt[3]{8} \cdot \sqrt[3]{2}=2 \sqrt[3]{2}
\end{aligned}
$$

Splitting Quotients

$$
\sqrt[3]{\frac{x^{3}}{y^{3}}}=\frac{\sqrt[3]{x^{3}}}{\sqrt[3]{y^{3}}}=\frac{x}{y}
$$

$$
\sqrt[3]{\frac{27}{125}}=\frac{\sqrt[3]{27}}{\sqrt[3]{125}}=\frac{3}{5}
$$

## More Examples: Exponents and Roots

Find the length of the hypotenuse of the right triangle.


# Undefined Value Over the Set of Real Numbers 

## Undefined Value Over the Set of Real Numbers

There are two types of expressions that are undefined over the set of real numbers:

- Fractions with zero in the denominator (or an expression equivalent to zero)

$$
\text { Examples: } \frac{-3}{0} ; \frac{0}{0} ; \frac{x-3}{x+3} \text {, where } x=-3
$$

-Square roots of negative numbers (or expressions which, when simplified, result in negative numbers).

Examples: $\sqrt{-1} ; x^{2}+1=0 ; \sqrt{-3 x^{2}} ; \sqrt{x^{3}-2}$, where $x=-1$

## The Incredible Zero

## The Incredible Zero

-It is unique in representing nothingness.
-As a placeholder it gives our number system its power.
-It acquires different meaning based on its location. Think 30 versus 3,000.


The Origin of the Number Zero
http://www.smithsonianmag.com /history/origin-number-zero180953392/\#qagAYijydW3RXhh k. 99

## Properties of Zero

| Property | Example |
| :--- | :--- |
| $a+0=a$ | $4+0=4$ |
| $a-0=a$ | $4-0=4$ |
| $a \times 0=0$ | $6 \times 0=0$ |
| $0 / a=0$ | $0 / 3=0$ |
| $a / 0=$ undefined (dividing by zero is undefined) | $7 / 0=$ undefined |
| $0^{a}=0$ (a is positive) | $0^{4}=0$ |
| $a^{0}=1$ | $7^{0}=1$ |

http://www.mathsisfun.com/numbers/zero.html

## The Problem with Zero



You can express a fraction with 0 in the denominator, but it has no meaning.

Division by zero is undefined. Mathematicians have never defined the meaning because there is no good definition.

## How many times can you throw nothing into no baskets?

## As many times as you want. It's just not a real number.



## To learn more: <br> https://www.youtube.com/watch?v=N KmGVE85GUU

## Imaginary Numbers

## Imaginary Numbers?

Try squaring numbers to see if we can get a negative result.

$$
1^{2}=1 \quad 0^{2}=0 \quad(-2)^{2}=4 \quad(0.2)^{2}
$$

"Imagine" there is such a number. Let's call this number $\boldsymbol{i}$ for imaginary. Then we can do this...

$$
\boldsymbol{i}^{2}=\boldsymbol{i} \cdot \boldsymbol{i}=-\mathbf{1}
$$

"Imagine" there is such a number, called $i$ for imaginary. Then we can also do this...

$$
\begin{gathered}
\sqrt{i^{2}}=\sqrt{-1} \\
i=\sqrt{-1}
\end{gathered}
$$

## Are imaginary numbers truly imaginary?

There was a point in time when they were thought to be impossible. But then people researched them more and discovered they were actually useful and important because they filled a gap in mathematics (but the word "imaginary" stuck).

The true power of imaginary number comes when combined with real numbers. This gave birth to a whole new mathematics...


$$
\begin{array}{ccc}
-1+6 i & & 7-5 i \\
& 2+3 i & \\
& & \text { yoưnspire } \\
& \text { GRDD }
\end{array}
$$

## Do imaginary numbers serve any purpose?


https://youtu.be/tSamA58MhQ8
https://youtu.be/fFuVJd36iKg

## 3-Dimensional Shapes

Performance Gap 3


## Volume of a Cylinder

Find the volume of the pizza below.

Thickness $=\mathbf{a}$ [

From the Formula Sheet:
Radius $=\mathbf{z}$

$$
V=\pi r^{2} h
$$

Substitute given information.

$$
V=\pi z^{2} a
$$

Notice anything?

$$
V=\operatorname{pi}(z \cdot z) a
$$

This is why it's called pizza.

## Anchor Chart: Surface Area of Right

 Triangular Prisms$$
\mathrm{SA}=p h+2 \mathrm{~B}
$$



1 Perimeter of the Base and Height
(2) Area of the Base

B


## $p$ and $h$

$$
\begin{aligned}
& p=(3+4+5) \\
& p=12 \\
& h=1
\end{aligned}
$$



$$
\begin{aligned}
& \mathrm{B}=\frac{1}{2} b h \\
& \mathrm{~B}=\frac{1}{2}(3 \cdot 4) \\
& \mathrm{B}=\frac{1}{2}(12)=6
\end{aligned}
$$

(3) Solve

$$
\begin{array}{ll}
S A=p h+2 B & S A=12 \\
S A=(12)(1)+2(6) & S A=24
\end{array}
$$

## Examining a Right Triangular Prism



## Build Your Own Right Triangular Prism



1. Enter your name or a proxy name.
2. Tap on "START."
3. Follow the instructions to build the right triangular prism.
4. Answer the questions that follow.

GeoGebra

| ${ }^{1: 91}$ | .1. 56 ( 6 |
| :---: | :---: |
| GesGebra | 2 |

The applet below lets you build a right triangular prism with the specifications listed.

## Explore!

Use the applet below to build a right triangular prism with the following dimensions:

## Prism Height $=5$

Base of Triangle $=4$
Height of Triangle $=3$
One side of triangle $=5$

Find the the Total Surface Area, Lateral Area and Volume of the right triangular prism by answering the questions below.

Task 1


## Build and Explore Your Own Solids

There are several other lessons/activities for various solids such as pyramids, cylinders, prisms, cones, and spheres, developed by other teachers and available under Classroom Resources.


## GeoGebra's Augmented Reality Feature



# Solving Algebraic Inequalities 

Performance Gap 5

## Focusing on High Impact Indicators Inequalities

A.3.a Solve linear inequalities in one variable with rational number coefficients.
A.3.b Identify or graph the solution to a one variable linear inequality on a number line.
A.3.c Solve real-world problems involving inequalities.
A.3.d Write linear
inequalities in one variable to represent context.

## Solve Real World Problems with Inequalities

Annie is planning a business meeting for her company. She has a budget of $\$ 1,325$ for renting a meeting room at a local hotel and providing lunch. She expects 26 people to attend the meeting. The cost of renting the meeting room is $\$ 270$. Write an inequality to show how to find the amount, $x$, Annie can spend on lunch for each person?

Establish the relationship:
Cost $\leq$ Budget or Budget $\geq$ Cost

$$
\begin{aligned}
& \text { Budget }=\$ 1,325 \\
& \text { Cost }=26 x+\$ 270
\end{aligned}
$$

Set-up the inequality.
Cost $\leq$ Budget

$$
\$ 26 x+\$ 270 \leq \$ 1,325
$$

## Solving Equations vs. Inequalities

| $3 x+15$ | $=24$ |
| ---: | :--- |
| $3 x+15-15$ | $=24-15$ |
| $\frac{3 x}{\not 2}$ | $=\frac{9}{3}$ |
| $x$ | $=3$ |





## With Only One Exception...

When multiplying or dividing both sides of the inequality with a negative number, the inequality sign must be reversed for the solution to remain true.

For example:

$$
\begin{aligned}
-3 x-15 & \geq 24 \\
-3 x-15+15 & \geq 24+15 \\
-3 x & \geq 39 \\
\frac{-3 x}{-3} & \leq \frac{39}{-3} \\
x \leq & \leq 13
\end{aligned}
$$



## The Million Dollar Question


yoûnspire $\underset{\text { GR }}{\text { Gen }}$

## Slopes and Graphing Linear Equations

Performance Gap 6


## Focusing on Slopes and Graphing Linear Equations



## The Concept of Slope



Workbook P. 8

## Is <br> $\mathrm{SlO}_{\mathrm{p}^{6}}$ <br> 



This Photo by Unknown Author is licensed under CC BY-NC-ND

## Forms of Linear Equations

## Forms of Linear Equations

## Equations

## Slope-Intercept Form

Point-Slope Form
Standard Form
$y=m x+b$

$$
\begin{gathered}
y-y_{1}=m\left(x-x_{1}\right) \\
c x+d y=e
\end{gathered}
$$

$m=$ slope
$b=y$-intercept
$\left(x_{1}, y_{1}\right)=a$ point on the line
$c, d$ and $e$ are constants


Workbook P. 8

## Graphing Linear Equations


https://www.geogebra.org/m/n5gskda8

## Anchor Chart for Finding Slope

| T-Chart | Slope-Intercept | Standard | Graph |
| :---: | :---: | :---: | :---: |
| Use the slope formula. $m=\frac{\left(y_{2}-y_{1}\right)}{\left(x_{2}-x_{1}\right)}$ <br> Example: | Locate $m$ in the equation. $y=m x+b$ | $c x+d y=e$ <br> Transform equation to slope-intercept form and locate $m$ in the equation. |  |
| $\boldsymbol{x}$ $\boldsymbol{y}$ <br> 1 -9 <br> 3 -6 <br> 5 -3$\begin{aligned} & m=\frac{-3-(-9)}{5-1} \\ & m=\frac{6}{4}=\frac{3}{2} \end{aligned}$ | $\begin{aligned} & y=3 x-4 \\ & y=m x+b \\ & m=3 \end{aligned}$ | $\begin{aligned} & 3 x+9 y=4 \\ &-3 x \quad-3 x \\ & \hline 9 y=-3 x+4 \\ & \frac{9 y}{9}=\frac{-3 x}{9}+\frac{4}{9} \\ & y=\frac{-3}{9} x+\frac{4}{9} \\ & m=\frac{-3}{9} \end{aligned}$ | Locate two points on the graph, then use the slope formula. <br> Example: $\begin{aligned} & (0.2) \operatorname{and}(2,3) \\ & m=\frac{\left(y_{2}-y_{1}\right)}{\left(x_{2}-x_{1}\right)} \\ & m=\frac{3-2}{2-0} \\ & m=\frac{1}{2} \end{aligned}$ |

## Graphing Exercise

## (1) neorpod



Join at join.nearpod.com or in the app

younspire

## Supplemental Graphing Activities

What's green and fluffy and comes from outer space?


Garefuly graph aach tive 54 match formbers wich betters. Werito sham it the spacel belame

| $2 x-4 y-3$ | $2 e+8 y=-4$ | -2x+4y-1 | $3 x+3 y=-15$ | $15-3 y+3$ | $\mathrm{r}=-\frac{8}{}$ | $x+3 \mathrm{c}=18$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| As - $4 \mathrm{r}=-4.4$ |  | In - $100-20$ |  | bx $-3 \mathrm{y}=13$ | $2 \mathrm{x}+4 \times 23$ | * $+i r+27$ |

Answer:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 23 | 15 | 0 | 22 | 2 | 5 | 12 | 19 |  |  |  |  |  |

D. Wekselgreene (2010). Some fun(ish) worksheets. http://exponentialcurve.blogspot.com/2010/04/some-funishworksheets.html

## Anchor Chart for Using Slope to Interpret Distance vs. Time Graphs



## Linear Modeling Word Problem

Mathematical Reasoning

## Answer Explanation Calculator

$A$ Flag for Review

A scientist is studying red maple tree growth in a state park. She measured the trunk diameters of a sample of trees in the same month every other year. The tables show the data for two of the trees.

Tree 1

| Year | Trunk <br> Diameter <br> (inches) |
| ---: | :---: |
| 1 | 18.6 |
| 3 | 19.2 |
| 5 | 19.8 |
| 7 | 20.4 |
| 9 | 21.0 |
| 11 | 21.6 |
| 13 | 22.2 |

$$
\begin{gathered}
m=\frac{\left(y_{2}-y_{1}\right)}{\left(x_{2}-x_{1}\right)} \\
m=\frac{(19.2-18.6)}{(3-1)} \\
m=\frac{(0.6)}{(2)} \\
m=0.3 \\
y=m x+b \\
18.6=0.3(1)+b \\
18.6-0.3=b \\
18.3=b
\end{gathered}
$$

This is the final year in which she will collect data. When her data collection is complete, she will predict future red maple tree growth.

## Formula Sheet

Calculator Reference

The scientist creates an equation that models her data for each tree so that she can predict the diameter in the future. Complete a linear equations that fits the data for tree 1, where $x$ is the year and $y$ is the trunk diameter, in inches.

Click on the variables and number you want to select and drag them into the boxes.

18.0


## Linear Modeling Word Problem

Mathematical Reasoning
를 Question 6 of 10

## Answer Explanation E Calculator

$A$ Flag for Review

A scientist is studying red maple tree growth in a state park. She measured the trunk diameters of a sample of trees in the same month every other year. The tables show the data for two of the trees.


$$
\begin{aligned}
m & =\frac{0.6}{2} \\
m & =0.3
\end{aligned}
$$

The y-intercept is simply the trunk diameter at year 0 .

$$
\begin{gathered}
b=18.6-0.3 \\
b=18.3
\end{gathered}
$$

This is the final year in which she will collect data. When her data collection is complete, she will predict future red maple tree growth.

AFormula Sheet
Calculator Reference

The scientist creates an equation that models her data for each tree so that she can predict the diameter in the future. Complete a linear equations that fits the data for tree 1, where $x$ is the year and $y$ is the trunk diameter, in inches.

Click on the variables and number you want to select and drag them into the boxes.

18.0



TESTING SERVICE ${ }^{\circ}$

## Some people want it to happen, some wish it would happen, others make it happen.

## Questions



## Session Survey

Your feedback is important. Please scan the QR code below to rate this session.

QR CODE PLACEHOLDER


## Thank you!



Ronald Cruz<br>GED ${ }^{\circledR}$ National Trainer rcruz@bucketPD.com

Communicate with GED Testing Service ${ }^{\circledR}$ communications@ged.com

