younspire 2023 GED CONFERENCE

Grasping GED[®] Higher Order Math Concepts for Deeper Understanding







GED TESTING SERVICE® Facilitator

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In this session, we will

- Discuss how to push high achieving students to score at a college ready or college ready + credit level on the mathematical reasoning test.
- Focus on critical performance gaps based on field testing data
- Explore strategies, resources and ideas to address these performance gaps.







Three Score Level Indicators on GED Ready®





How do we make high performing GED students perform even better?



Performance Gaps

- 1. Non-Calculator Items
- 2. Exponents and Roots
- 3. Three-Dimensional Shapes
- 4. Algebraic Computations
- 5. Inequalities
- 6. Slope and Graphing Linear Equations
- 7. Multiple Correct Answers

Tuesdays for Teachers: GED Knowledge & Skill Gaps - Math Session 1 – Oct. 26, 2021 Session 2 – Nov. 16, 2021



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Calculator-Prohibited Indicators

Performance Gap 1





Non-Calculator Items

- 1. Ordering Rational Numbers
- 2. Factors and Multiples
- 3. Distance on a Number Line
- 4. Operations on Rational Numbers
- 5. Rules of Exponents



- 7. Cubes and Cube Roots of Rational Numbers
- 8. Undefined Value Over the Set of Real Numbers





60 Second Challenge: Ordering Rational Numbers

$$\frac{5}{8}, -3, 0.8314, \frac{1}{16}, -\pi, 0.4823, \frac{5}{12}$$



Let's Try Again





Ordering Rational Numbers: Use a Number Line

$$\frac{5}{8}$$
, -3, 0.8314, $\frac{1}{16}$, - π , 0.4823, $\frac{5}{12}$



- 1. Draw a number line and mark benchmark numbers.
- 2. Plot the easiest given numbers in reference to benchmark numbers.
- 3. Compare and plot the last remaining numbers.



Factors and Multiples

•Greatest Common Factor – Used to simplify fractions

Example:
$$\frac{22}{12} = \frac{22 \div 2}{12 \div 2} = \frac{11}{6}$$

•Least Common Multiple (LCM) – Used to add/subtract fractions with unlike denominators.

Example:
$$\frac{7}{6} - \frac{1}{4} = \frac{14 - 3}{12} = \frac{11}{12}$$



Greatest Common Factor

Example 1: Find the GCF of 35 and 42.



Example 2: Find the GCF of 18 and 24.





Least Common Multiple (LCM)

Example 1: Find the LCM of 8, 4, and 6.

Method 1: Listing Multiples

$$8 \rightarrow 8$$
, 16, 24, 32, 40, 48
4 $\rightarrow 4$, 8, 12, 16, 20, 24, 28, 32
6 $\rightarrow 6$, 12, 18, 24, 30, 36

LCM = 24

Example 2: Find the LCM that is necessary to perform the indicated operation: $\frac{3}{8} - \frac{1}{42} =$





Distance on a Number Line

The distance is how far apart points are on a number line. 5 units





Find the distance between -24 and 13 on a number line.



Distance on a Number Line

The distance between two points A and B on a number line is:

$$= |A - B|$$

Example:

Find the distance between -24 and 13 on a number line.

$$= |-24 - 13|$$

 $= |-37|$
 $= 37$

Absolute Value is the distance of a number from zero on a number line.



Operations on Rational Numbers

By MathTricks on Facebook Reels https://www.facebook.com/reel/657776426 087451/





Fractions | Adding and subtracting fractions|Dividing and multiply fractions #fraction #reels

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Exponents and Roots/Radicals

Performance Gap 2



finally found the square root!



Rules of Exponents

Workbook P. 3

Name	Rule	Example
Product	$a^m \cdot a^n = a^{m+n}$	$x^3 \cdot x^4 = x^{3+4} = x^7$
Quotient	$a^m \div a^n = a^{m-n}$	$p^5 \div p^2 = p^{5-2} = p^3$
Power of a Power	$(a^m)^n = a^{mn}$	$(z^3)^2 = z^{3 \cdot 2} = z^6$
Power of a Product	$(ab)^m = a^m b^m$	$(3y)^2 = 3^2 y^2 = 9y^2$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{5}{3}\right)^2 = \frac{5^2}{3^2} = \frac{25}{9}$
Zero Exponent	$a^0 = 1$	$x^0 = 1; 6^0 = 1; 0^0 = 1$
Negative Exponent	$a^{-m} = \frac{1}{a^m}$	$b^{-3} = \frac{1}{b^3}; 5^{-2} = \frac{1}{5^2}$
Fractional Exponent	$a^{\frac{m}{n}} = \sqrt[n]{a^m}$	$4^{\frac{3}{2}} = \sqrt[2]{4^3} = \sqrt{64} = 8$

Square and Square Root Tricks (Part 1)

Combining of Similar Radicals

$$a\sqrt{b} + a\sqrt{b} = (a + a)\sqrt{b}$$
$$a\sqrt{b} - c\sqrt{b} = (a - c)\sqrt{b}$$

Example 1:
$$2\sqrt{5} + 6\sqrt{5} = (2+6)\sqrt{5} = 8\sqrt{5}$$

Example 2: $3\sqrt{2} - 5\sqrt{2} = (3-5)\sqrt{2} = -2\sqrt{2}$



Square and Square Root Tricks (Part 2)

Splitting Products

$$\sqrt{x^3} = \sqrt{x^2 \cdot x} = \sqrt{x^2}\sqrt{x} = |x|\sqrt{x}$$
$$\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = |2|\sqrt{5}$$

Splitting Quotients

$$\sqrt{\frac{x^2}{y^2}} = \frac{\sqrt{x^2}}{\sqrt{y^2}} = \frac{x}{y}$$

$$\sqrt{\frac{4}{25}} = \frac{\sqrt{4}}{\sqrt{25}} = \frac{|2|}{|5|}$$





Square and Square Root Exercise

Workbook P. 3

Simplify $2\sqrt{2}(2\sqrt{3} + 3\sqrt{3})$ = $2\sqrt{2}(5\sqrt{3}) = 10\sqrt{2 \cdot 3} = 10\sqrt{6}$

Simplify $3\sqrt{24x^3}$

 $= 3\sqrt{4 \cdot 6 \cdot x^{2} \cdot x} = 3 \cdot 2 \cdot x\sqrt{6x} = 6x\sqrt{6x}$ Simplify $(-4\sqrt{2})^{2}$ $= (-4)^{2} (\sqrt{2})^{2} = 16 \cdot 2 = 32$ Simplify $\sqrt{\frac{12x^{2}}{4}} = \frac{\sqrt{12x^{2}}}{\sqrt{4}} = \frac{\sqrt{4 \cdot 3 \cdot x^{2}}}{2} = \frac{2x\sqrt{3}}{2} = x\sqrt{3}$

Cube and Cube Root Tricks (Part 1)

Combining of Similar Radicals

$$a\sqrt[3]{b} + a\sqrt[3]{b} = (a+a)\sqrt[3]{b}$$
$$a\sqrt[3]{b} - c\sqrt[3]{b} = (a-c)\sqrt[3]{b}$$

Example 1:
$$2\sqrt[3]{5} + 6\sqrt[3]{5} = (2+6)\sqrt[3]{5} = 8\sqrt[3]{5}$$

Example 2: $3\sqrt[3]{2} - 5\sqrt[3]{2} = (3-5)\sqrt[3]{2} = -2\sqrt[3]{2}$



Cube and Cube Root Tricks (Part 2)

Splitting Products

$$\sqrt[3]{x^4} = \sqrt[3]{x^3 \cdot x} = \sqrt[3]{x^3} \cdot \sqrt[3]{x} = x\sqrt[3]{x}$$
$$\sqrt[3]{16} = \sqrt[3]{8 \cdot 2} = \sqrt[3]{8} \cdot \sqrt[3]{2} = 2\sqrt[3]{2}$$

Splitting Quotients

$$\sqrt[3]{\frac{x^{3}}{y^{3}}} = \frac{\sqrt[3]{x^{3}}}{\sqrt[3]{y^{3}}} = \frac{x}{y}$$

$$\sqrt[3]{\frac{27}{125}} = \frac{\sqrt[3]{27}}{\sqrt[3]{125}} = \frac{3}{5}$$



More Examples: Exponents and Roots

1. Find the length of the hypotenuse of the right triangle.



Undefined Value Over the Set of Real Numbers



Undefined Value Over the Set of Real Numbers

There are two types of expressions that are undefined over the set of real numbers:

 Fractions with zero in the denominator (or an expression equivalent to zero)

Examples:
$$\frac{-3}{0}$$
; $\frac{0}{0}$; $\frac{x-3}{x+3}$, where $x = -3$

 Square roots of negative numbers (or expressions which, when simplified, result in negative numbers).

Examples: $\sqrt{-1}$; $x^2 + 1 = 0$; $\sqrt{-3x^2}$; $\sqrt{x^3 - 2}$, where x = -1



The Incredible Zero





The Incredible Zero

- It is unique in representing nothingness.
- As a placeholder it gives our number system its power.
- It acquires different meaning based on its location. Think 30 versus 3,000.



The Origin of the Number Zero

http://www.smithsonianmag.com /history/origin-number-zero-180953392/#qagAYijydW3RXhh k.99



Properties of Zero

Property	Example
a + 0 = a	4 + 0 = 4
a – 0 = a	4 - 0 = 4
$a \times 0 = 0$	$6 \times 0 = 0$
0 / a = 0	0/3 = 0
a / 0 = undefined (<u>dividing by zero is undefined</u>)	7/0 = undefined
0 ^a = 0 (a is positive)	$0^4 = 0$
$a^0 = 1$	$7^0 = 1$

http://www.mathsisfun.com/numbers/zero.html



The Problem with Zero



You can express a fraction with 0 in the denominator, but it has no meaning. How many times can you throw nothing into no baskets?

Division by zero is undefined. Mathematicians have never defined the meaning because there is no good definition. As many times as you want. It's just not a real number.



To learn more: <u>https://www.youtube.com/watch?v=N</u> <u>KmGVE85GUU</u> younspire

Imaginary Numbers



Imaginary Numbers?

Try squaring numbers to see if we can get a negative result.

$$1^2 = 1$$
 $0^2 = 0$ $(-2)^2 = 4$ $(0.2)^2$

"**Imagine**" there is such a number. Let's call this number *i* for imaginary. Then we can do this...

$$i^2 = i \cdot i = -1$$

"**Imagine**" there is such a number, called *i* for imaginary. Then we can also do this...

$$\sqrt{i^2} = \sqrt{-1}$$
$$i = \sqrt{-1}$$



Are imaginary numbers truly imaginary?

There was a point in time when they were thought to be impossible. But then people researched them more and discovered they were actually useful and important because they filled a gap in mathematics (but the word "imaginary" stuck).

The true power of imaginary number comes when combined with real numbers. This gave birth to a whole new mathematics...



Do imaginary numbers serve any purpose?



https://youtu.be/tSamA58MhQ8



https://youtu.be/fFuVJd36iKg

3-Dimensional Shapes

Performance Gap 3



Volume of a Cylinder

Find the volume of the pizza below.



From the Formula Sheet:

$$V = \pi r^2 h$$

Substitute given information.

$$V = \pi z^2 a$$
 Notice anything?

$$V = pi(z \cdot z)a$$
 This is why it's called pizza.



Radius = z



Examining a Right Triangular Prism







Build Your Own Right Triangular Prism

GeoGebra



- 1. Enter your name or a proxy name.
- 2. Tap on "START."
- 3. Follow the
 - instructions to build the right triangular prism.
- 4. Answer the questions that follow.

1:26 JI 5G 72	1:41	5G 70
	< Camera	
	Ge@Gebra	B
	The applet below lets you build a right triangular with the specifications listed.	r prism
	Explore!	
	Use the applet below to build a right triangular p with the following dimensions:	orism
Ge@Gebra Classroom	Prism Height = 5 Base of Triangle = 4	
Welcome to COABE 2024: Build Your Own Right Triangular Prism	Height of Triangle = 3 One side of triangle = 5	
Join with your GeoGebra account and continue your work any time.	Find the the Total Surface Area, Lateral Area Volume of the right triangular prism by answe questions below.	and ering the
SIGN IN	Task 1	
OR	Prism Height = 5	
Name	5 Base of Triangle	=6
START	Height of Triangle	e =7
	7,32 One Side of Triat	ngle = 8 Right or Acute
	5 6 Blue Angle = 5 6 Show Bases	Obtuse
AA	Show Lateral	Area Show TSA
	Filling Show LA	Show V
	🔒 geogebra.org	

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Build and Explore Your Own Solids

There are several other lessons/activities for various solids such as pyramids, cylinders, prisms, cones, and spheres, developed by other teachers and available under Classroom Resources.







GeoGebra's Augmented Reality Feature

















Solving Algebraic Inequalities

Performance Gap 5







Solve Real World Problems with Inequalities

Annie is planning a business meeting for her company. She has a budget of \$1,325 for renting a meeting room at a local hotel and providing lunch. She expects 26 people to attend the meeting. The cost of renting the meeting room is \$270. Write an inequality to show how to find the amount, x, Annie can spend on lunch for each person?



- Establish the relationship:
- $Cost \leq Budget \text{ or } Budget \geq Cost$
 - Budget = \$1,325
 - Cost = 26x + \$270

Set-up the inequality. Cost ≤ Budget

 $26x + 270 \le 1,325$



Solving Equations vs. Inequalities







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With Only One Exception...

When multiplying or dividing both sides of the inequality with a negative number, the inequality sign must be reversed for the solution to remain true.



The Million Dollar Question







Slopes and Graphing Linear Equations

Performance Gap 6





Focusing on Slopes and Graphing Linear Equations









Forms of Linear Equations

Forms of Linear Equations	Equations
Slope-Intercept Form	y = mx + b
Point-Slope Form	$y - y_1 = m(x - x_1)$
Standard Form	cx + dy = e

m = slope

b = y-intercept

 $(x_1, y_1) = a$ point on the line

c, d and e are constants



Graphing Linear Equations





https://www.geogebra.org/m/n5gskda8



Anchor Chart for Finding Slope

T-Chart	Slope-Intercept	Standard	Graph
Use the slope formula. $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$ Example: $\boxed{\begin{array}{c c} \mathbf{x} & \mathbf{y} \\ 1 & -9 \\ 3 & -6 \\ 5 & -3 \end{array}}$ $m = \frac{-3 - (-9)}{5 - 1}$ $m = \frac{6}{4} = \frac{3}{2}$	Locate <i>m</i> in the equation. y = mx + b Example: y = 3x - 4 y = mx + b m = 3	cx + dy = e Transform equation to slope-intercept form and locate <i>m</i> in the equation. Example: 3x + 9y = 4 -3x - 3x 9y = -3x + 4 $\frac{9y}{9} = \frac{-3x}{9} + \frac{4}{9}$ $y = \frac{-3}{9}x + \frac{4}{9}$ $m = \frac{-3}{9}x + \frac{4}{9}$	$ \begin{aligned} \hline y \\ \hline y \\ $
		$m - \frac{1}{9}$	$m = \frac{1}{2}$

Graphing Exercise



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Supplemental Graphing Activities





Each person chooses a color and draws three ships like those on the left. Ships may be placed horizontally or vertically but not diagonally.

To get a hit, write an equation in slope-intercept form that crosses through a point that an enemy ship occupies. You may not use vertical or horizontal lines.

Graph the equation. Record your move on the chart.

You may hit more than one ship with a single line if you are clever! Mark an X on any points you've crossed. You cannot hit your own ship.

Created by iisanumber.blogspot.com

The winner sinks all of his opponent's boats first.

Kathryn (2013). Slope-Intercept Form Battleship. i is a number. http://iisanumber.blogspot.com/2013/02/slope-intercept-formbattleship.html Algebra I Unit 7, Lesson 8: Individual Work Name

ot 7, Lesson a. mainiaan work

What's green and fluffy and comes from outer space?

Period



Carefully graph each line to match numbers with letters. Write them in the spaces below.

$\ge x-4\gamma = 0$	2e + 8y = -64	$\cdot \ge x + \alpha_2 = 0$	3x + 5y = -15	175 - 3y = 5	72.4	x + 3p = 18
.4s =6y = .54	4x - y 7 - 8	$3\pi-10\gamma=20$	$\Delta x=\dot{\Psi}_{y}=\bar{x}$	$\delta x - 3 y = 12$	$R_X + 4\gamma = 43$	$a+3\gamma=27$

Answers



D. Wekselgreene (2010). Some fun(ish) worksheets. http://exponentialcurve.blogspot.com/2010/04/some-funishworksheets.html

Anchor Chart for Using Slope to Interpret **Distance vs. Time Graphs**



Example:

Maria walked to the store at the end of her street. bought a gallon on milk and then ran all the way back.

 Time	Distance	
0	0	
1	20	
2	40	
3	40	
4	40	
5	0	

Time	Distance
0	0
1	20
2	40
3	40
4	80
5	120

Time

Lucy walked slowly along

the road, stopped to look

at her cell phone,

running.

realized that she was

late, and then started



Scenario 3

Example:

Opposite Tom's home is a hill. He climbed slowly up the hill, walked across the top, and then ran quickly down the other side.

Time	Distance
0	0
1	10
2	20
3	40
4	60
5	120



Scenario 4

Example:

Mario went out to walk with some friends. Upon realizing he left his wallet, ran back home to get it. He then had to run to catch up with the others.

Time	Distance
 0	0
1	30
2	60
3	0
4	60
5	120

Linear Modeling Word Problem

Mathematical Reasoning

Answer Explanation 🖯 Calculator

A scientist is studying red maple tree growth in a state park. She measured the trunk diameters of a sample of trees in the same month every other year. The tables show the data for two of the trees.

Tree 1	
Year	Trunk Diameter (inches)
1	18.6
3	19.2
5	19.8
7	20.4
9	21.0
11	21.6
13	22.2

This is the final year in which she will collect data. When her data collection is complete, she will predict future red maple tree growth.

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$
$$m = \frac{(19.2 - 18.6)}{(3 - 1)}$$
$$m = \frac{(0.6)}{(2)}$$
$$m = 0.3$$
$$y = mx + b$$
$$18.6 = 0.3(1) + b$$
$$18.6 - 0.3 = b$$

Formula Sheet

A Calculator Reference

The scientist creates an equation that models her data for each tree so that she can predict the diameter in the future. Complete a linear equations that fits the data for tree 1, where x is the year and y is the trunk diameter, in inches.

Click on the variables and number you want to select and drag them into the boxes.







Linear Modeling Word Problem

Mathematical Reasoning

Answer Explanation Calculator

A scientist is studying red maple tree growth in a state park. She measured the trunk diameters of a sample of trees in the same month every other year. The tables show the data for two of the trees.



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Click on the variables and number you want to select and drag them into the boxes.

☐ Formula Sheet







A Calculator Reference

Final Words





6 6 Some people want it to happen, some wish it would happen, others make it happen.

Michael Jordan



Questions



Session Survey

Your feedback is important. Please scan the QR code below to rate this session.

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QR CODE PLACEHOLDER

Thank you!



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