

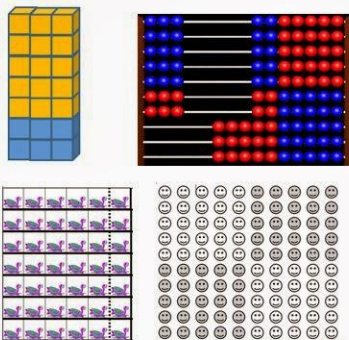
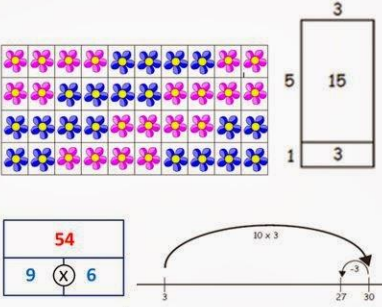
Meeting the Challenges to Finish Strong in Math

Information, Resources, and Strategies for the Classroom

Mellissa Hultstrand – Mellissa.Hultstrand@GED.com

Debi K. Faucette – Debi.Faucette@GED.com

C-R-A – Essential for Understanding

Concrete	Representational	Abstract																									
<p>Students manipulate hands-on, concrete materials</p>	<p>Students draw and observe diagrams, or watch the teacher touching and moving hands-on materials</p>	<p>Numbers and mathematical symbols</p>																									
		<table border="1" data-bbox="1073 632 1360 720"> <thead> <tr> <th colspan="5">x 4 Patterns</th> </tr> </thead> <tbody> <tr> <td>4</td> <td>8</td> <td>12</td> <td>16</td> <td>20</td> </tr> <tr> <td>24</td> <td>28</td> <td>32</td> <td>36</td> <td>40</td> </tr> </tbody> </table> <table data-bbox="1057 751 1365 919"> <tbody> <tr> <td>8×5</td> <td>$45 \div 5$</td> </tr> <tr> <td>$(4 \times 2) \times 5$</td> <td>$(50 - 5) \div 5$</td> </tr> <tr> <td>$4 \times (2 \times 5)$</td> <td>$(50 \div 5) - (5 \div 5)$</td> </tr> <tr> <td>4×10</td> <td>$10 - 1$</td> </tr> <tr> <td>40</td> <td>9</td> </tr> </tbody> </table>	x 4 Patterns					4	8	12	16	20	24	28	32	36	40	8×5	$45 \div 5$	$(4 \times 2) \times 5$	$(50 - 5) \div 5$	$4 \times (2 \times 5)$	$(50 \div 5) - (5 \div 5)$	4×10	$10 - 1$	40	9
x 4 Patterns																											
4	8	12	16	20																							
24	28	32	36	40																							
8×5	$45 \div 5$																										
$(4 \times 2) \times 5$	$(50 - 5) \div 5$																										
$4 \times (2 \times 5)$	$(50 \div 5) - (5 \div 5)$																										
4×10	$10 - 1$																										
40	9																										

Fractions Tiles Template

Black Line Fraction Strips With Labels

--

	$\frac{1}{2}$		$\frac{2}{2}$
--	---------------	--	---------------

	$\frac{1}{3}$		$\frac{2}{3}$		$\frac{3}{3}$
--	---------------	--	---------------	--	---------------

	$\frac{1}{4}$		$\frac{2}{4}$		$\frac{3}{4}$		$\frac{4}{4}$
--	---------------	--	---------------	--	---------------	--	---------------

	$\frac{1}{5}$		$\frac{2}{5}$		$\frac{3}{5}$		$\frac{4}{5}$		$\frac{5}{5}$
--	---------------	--	---------------	--	---------------	--	---------------	--	---------------

	$\frac{1}{6}$		$\frac{2}{6}$		$\frac{3}{6}$		$\frac{4}{6}$		$\frac{5}{6}$		$\frac{6}{6}$
--	---------------	--	---------------	--	---------------	--	---------------	--	---------------	--	---------------

	$\frac{1}{7}$		$\frac{2}{7}$		$\frac{3}{7}$		$\frac{4}{7}$		$\frac{5}{7}$		$\frac{6}{7}$		$\frac{7}{7}$
--	---------------	--	---------------	--	---------------	--	---------------	--	---------------	--	---------------	--	---------------

	$\frac{1}{8}$		$\frac{2}{8}$		$\frac{3}{8}$		$\frac{4}{8}$		$\frac{5}{8}$		$\frac{6}{8}$		$\frac{7}{8}$		$\frac{8}{8}$
--	---------------	--	---------------	--	---------------	--	---------------	--	---------------	--	---------------	--	---------------	--	---------------

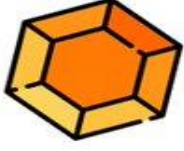
	$\frac{1}{9}$		$\frac{2}{9}$		$\frac{3}{9}$		$\frac{4}{9}$		$\frac{5}{9}$		$\frac{6}{9}$		$\frac{7}{9}$		$\frac{8}{9}$		$\frac{9}{9}$
--	---------------	--	---------------	--	---------------	--	---------------	--	---------------	--	---------------	--	---------------	--	---------------	--	---------------

	$\frac{1}{10}$		$\frac{2}{10}$		$\frac{3}{10}$		$\frac{4}{10}$		$\frac{5}{10}$		$\frac{6}{10}$		$\frac{7}{10}$		$\frac{8}{10}$		$\frac{9}{10}$		$\frac{10}{10}$
--	----------------	--	----------------	--	----------------	--	----------------	--	----------------	--	----------------	--	----------------	--	----------------	--	----------------	--	-----------------

	$\frac{1}{11}$		$\frac{2}{11}$		$\frac{3}{11}$		$\frac{4}{11}$		$\frac{5}{11}$		$\frac{6}{11}$		$\frac{7}{11}$		$\frac{8}{11}$		$\frac{9}{11}$		$\frac{10}{11}$		$\frac{11}{11}$
--	----------------	--	----------------	--	----------------	--	----------------	--	----------------	--	----------------	--	----------------	--	----------------	--	----------------	--	-----------------	--	-----------------

	$\frac{1}{12}$		$\frac{2}{12}$		$\frac{3}{12}$		$\frac{4}{12}$		$\frac{5}{12}$		$\frac{6}{12}$		$\frac{7}{12}$		$\frac{8}{12}$		$\frac{9}{12}$		$\frac{10}{12}$		$\frac{11}{12}$		$\frac{12}{12}$
--	----------------	--	----------------	--	----------------	--	----------------	--	----------------	--	----------------	--	----------------	--	----------------	--	----------------	--	-----------------	--	-----------------	--	-----------------

Free Math Worksheets at <http://www.math-drills.com>



Order of Operations



G **E** **M** **S**

Groupings

() { } []

Exponents

n^2

Multiply/Divide

Left to Right

\div / \times \cdot

Subtract/Add

Left to Right

$+$ $-$

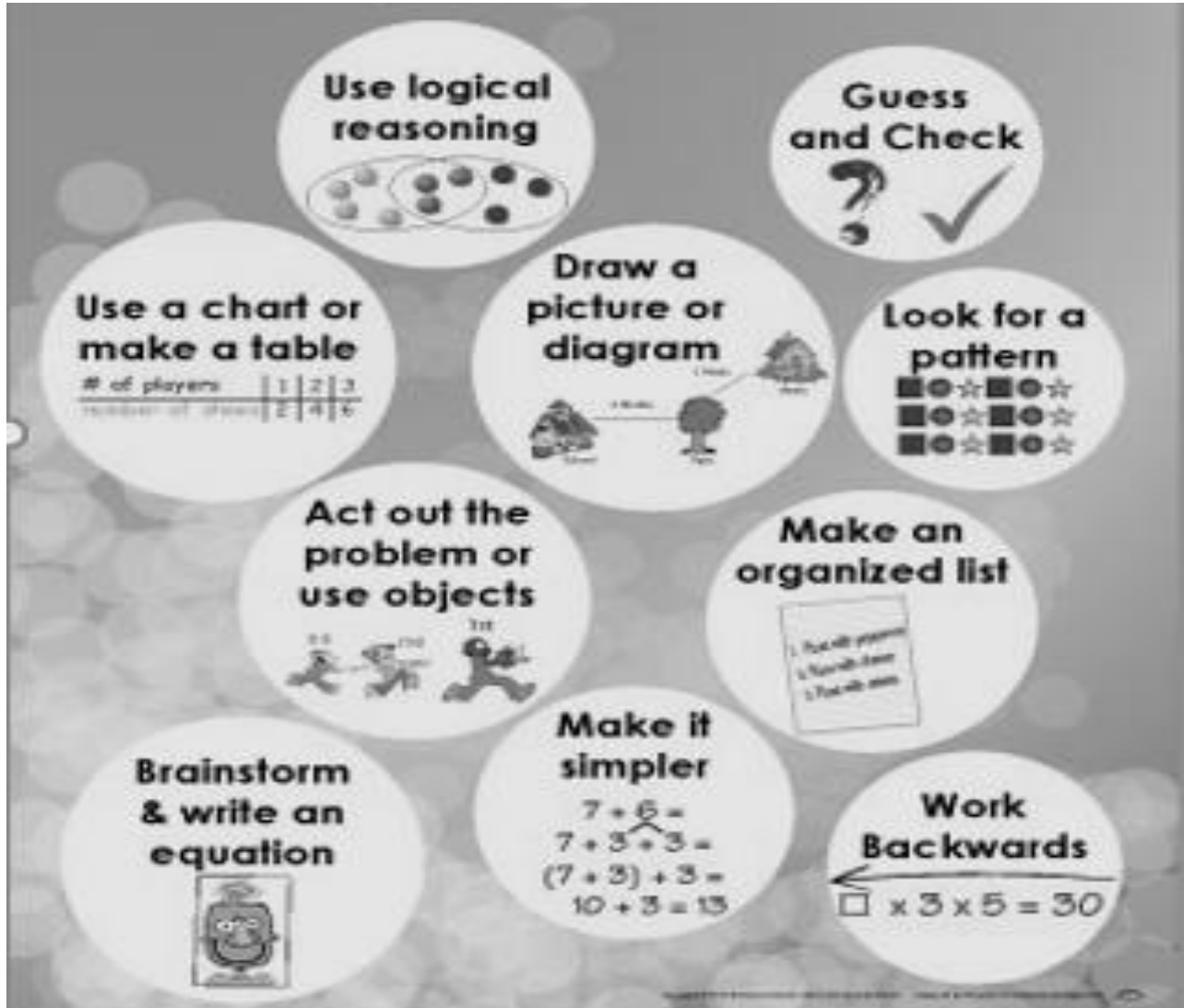
Reading and Reasoning Process

First Read: Read for Understanding

Second Read: Identify a Problem-Solving Process

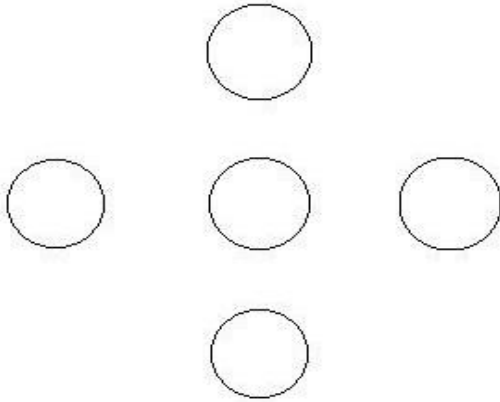
Third Read: Solve the Problem and Check for Reasonableness

Must Have Heuristics – Thinking Strategies



Guess and Check

Copy the figure below and place the digits 1, 2, 3, 4, and 5 in the circles so that sums across (horizontally) and down (vertically) are the same.



Draw a Diagram

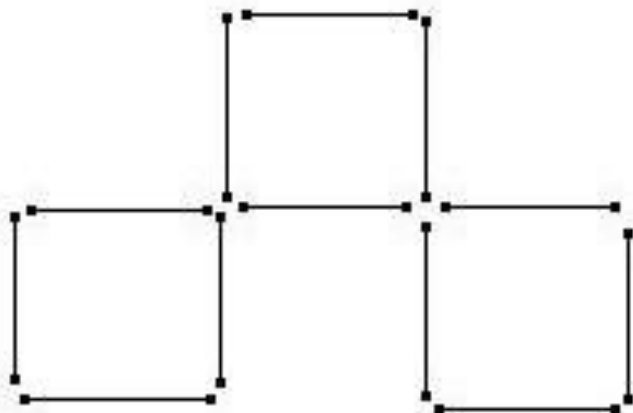
In a stock car race, the first five finishers in some order were a Ford, a Pontiac, a Chevrolet, a Buick, and a Dodge.

- The Ford finished seven seconds before the Chevrolet.
- The Pontiac finished six seconds after the Buick.
- The Dodge finished eight seconds after the Buick.
- The Chevrolet finished two seconds before the Pontiac.

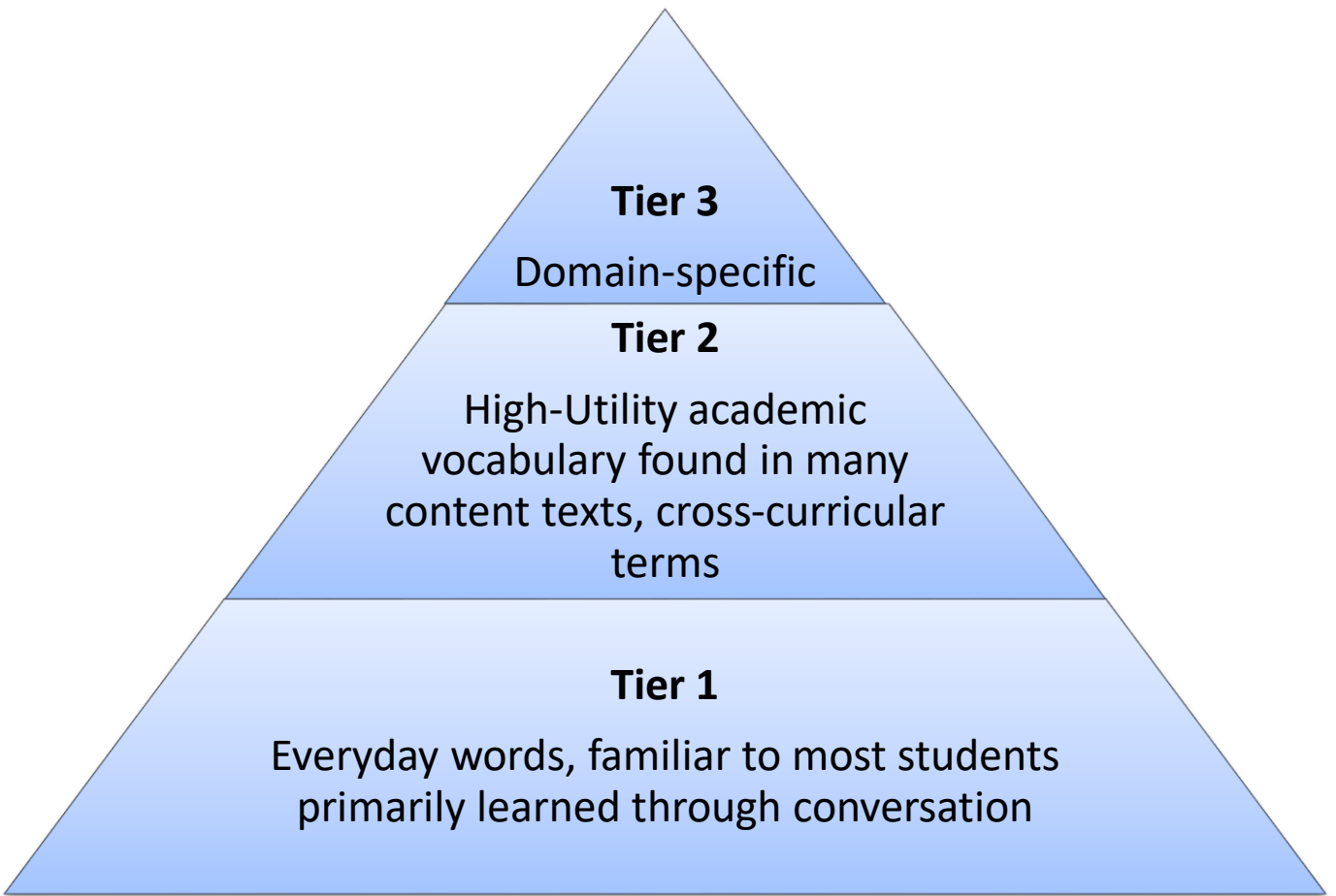
In what order did the cars finish the race? What strategy did you use?

Act It Out or Use Objects

The figure shows twelve toothpicks arranged to form three squares. How can you form five squares by moving only three toothpicks?



Tiered Vocabulary

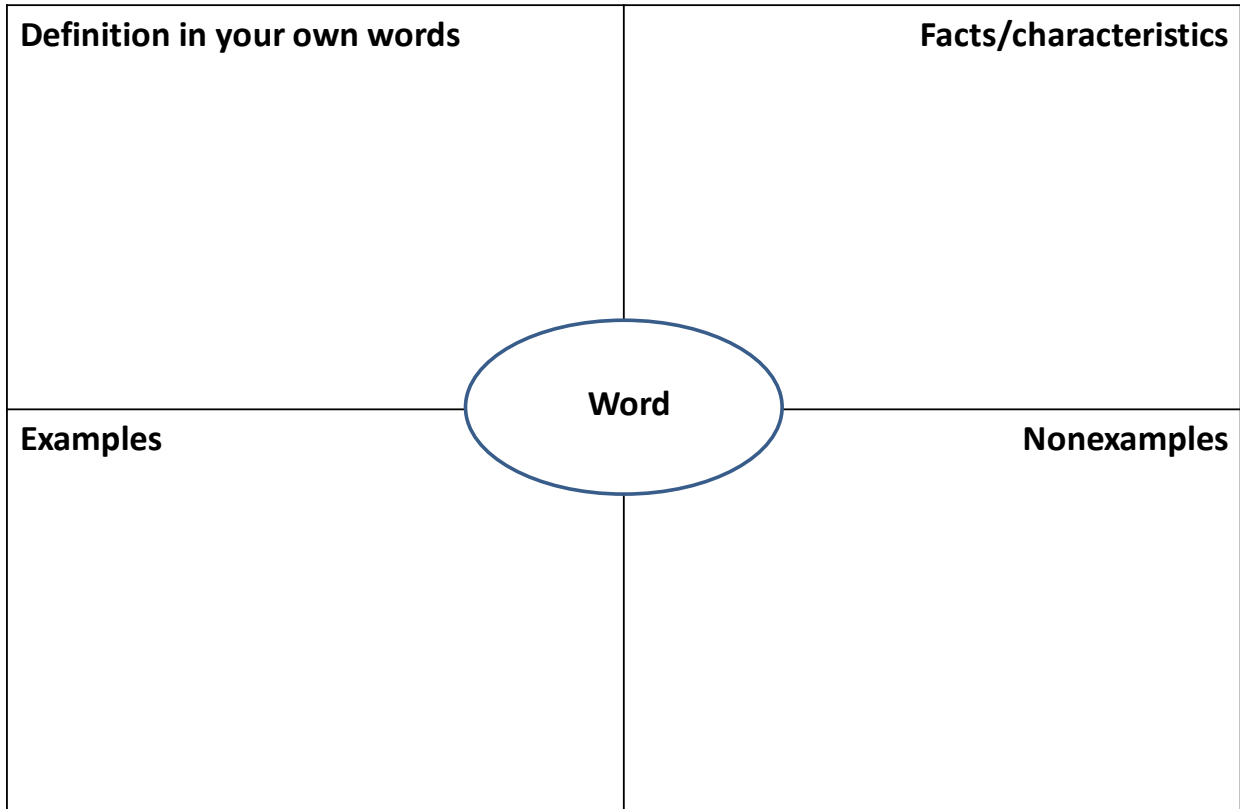


Frayer Model

Frayer Model

Definition in your own words	Facts/characteristics
Examples	Nonexamples

Word



How Good Are Your Visualization Skills?

Draw That!

1)

2)

3)

4)

5)

Can You Show that Algebraically?

Figure Out the Digits

1. $u \cdot r = z$
2. $t + w = t$
3. $r + r + r + r = z$
4. $x + y = q$
5. $s \cdot v = s$
6. $x^2 = q$
7. $r + r = u$
8. $x + u = s$
9. $\frac{y}{z} = \frac{x}{u}$

In this number puzzle, each letter (q-z) represents a different digit from 0-9. Find the correspondence between the letters and the digits. Be prepared to explain where you started, and the order in which you solved the puzzle.

Purposeful Questions

Question type	Description	Examples
Gathering information	Students recall facts, definitions, or procedures.	<ul style="list-style-type: none"> • When you write an equation, what does the equal sign tell you? • What is the formula for finding the area of a rectangle?
Probing thinking	Students explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the completion of a task.	<ul style="list-style-type: none"> • As you drew that number line, what decisions did you make so that you could represent $\frac{7}{4}$ on it? • Can you show and explain more about how you used a table to find the answer to the Smartphone Plans task?
Making the mathematics visible	Students discuss mathematical structures and make connections among mathematical ideas and relationships	<ul style="list-style-type: none"> • What does your equation have to do with the band concert situation? • How does that array relate to multiplication and division?
Encouraging reflection and justification	Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work.	<ul style="list-style-type: none"> • How might you prove that 51 is the solution? • How do you know that the sum of two odd numbers will always be even?

Math Translation Guide

The chart below gives you some of the terms that come up in a lot of word problems. Use them in order to translate or “set-up” word problems into equations.

English	Math	Example	Translation
What, a number	$x, n, \text{ etc.}$	Three more than a number is 8.	$N + 3 = 8$
Equivalent, equals, is, was, has, costs	$=$	Danny is 16 years old. A CD costs 15 dollars.	$d = 16$ $c = 15$
Is greater than Is less than At least, minimum At most, maximum	$>$ $<$ \geq \leq	Jenny has more money than Ben. Ashley’s age is less than Nick’s. There are at least 30 questions on the test. Sam can invite a maximum of 15 people to his party.	$j > b$ $a < n$ $t \geq 30$ $s \leq 15$
More, more than, greater, than, added to, total, sum, increased by, together	$+$	Kecia has 2 more video games than John. Kecia and John have a total of 11 video games.	$k = j + 2$ $k + j = 11$
Less than, smaller than, decreased by, difference, fewer	$-$	Jason has 3 fewer CDs than Carson. The difference between Jenny’s and Ben’s savings is \$75.	$j = c - 3$ $j - b = 75$
Of, times, product of, twice, double, triple, half of, quarter of	\times	Emma has twice as many books as Justin. Justin has half as many books as Emma.	$e = 2 \times j$ or $e = 2j$ $j = c \times \frac{1}{2}$ or $j = e/2$
Divided by, per, for, out of, ratio of ___ to ___	\div	Sophia has \$1 for every \$2 Daniel has. The ratio of Daniel’s savings to Sophia’s savings is 2 to 1.	$s = d \div 2$ or $s = d/2$ $d/s = 2/1$

Example 1

Jennifer has 10 fewer DVDs than Brad.

Step 1: j (has) = b (fewer) – 10

Remember, the word “has” is an equal sign and the word “fewer” is a minus sign, so:

Step 2: $j = b - 10$

Example 2

Clay got 1- fewer votes than Kimberly. Reuben got three times as many votes as Clay. The three contestants received a total of 90 votes. Write an equation in one variable that can be used to solve for the number of votes Kimberly received.

Step 1: Pick which unknown will be represented by the variable. Since you’re solving for Kimberly, let k be the number of votes Kimberly received.

Step 2: Represent the other two unknowns in terms of k . Clay got 10 fewer votes so it’s $k - 10$ and Reuben got three times that so it’s $3(k - 10)$.

Step 3: Set up the equation using all of the expressions to equal 90.

$$k + (k - 10) + 3(k - 10) = 90$$

Example 3:

A school is having a special even to honor successful alumni. The event will cost \$500, plus an additional \$85 for each alum who is honored. Write an equation that best represents the number of alumni that can be honored.

Step 1: The amount the school can spend is equal to or less than \$1,000, so it’s $\leq 1,000$

Step 2: The event has a fixed cost of \$500 and a variable of \$85 per alum so it’s $500 + 85a$.

Step 3: The equation then becomes $500 + 85a \leq 1,000$.

Example 4:

A computer repair company charges \$50 for a service call plus \$25 for each hour of work. Write an equation that represents the relationship between the bill, b , for a service call, and the number of hours spent on the call, h .

- Step 1: Some questions include a situation where there is more than one cost. One of them is fixed and one is variable. First identify the sum of the fixed and variable costs so b equals the total.
- Step 2: Next, identify the fixed cost of 50 and the variable cost of $25h$ (25 x the number of hours).
- Step 3: The equation then becomes $50 + 25h = b$.

Problem Solving Strategies from George Polya

George Polya (1887 – 1985) was one of the most famous mathematics educators of the 20th century (so famous that you probably never even heard of him). Dr. Polya strongly believed that the skill of problem solving could and should be taught – it is not something that you are born with. He identifies four principles that form the basis for any serious attempt at problem solving:

- 1. Understand the problem**
- 2. Devise a plan**
- 3. Carry out the plan**
- 4. Look back (reflect)**

1. Understand the problem

- What are you asked to find out or show?
- Can you draw a picture or diagram to help you understand the problem?
- Can you restate the problem in your own words?
- Can you work out some numerical examples that would help make the problem clearer?

2. Devise a plan

A partial list of Problem Solving Strategies includes:

- | | |
|----------------------------------|----------------------------------|
| <i>Guess and check</i> | <i>Solve a simpler problem</i> |
| <i>Make an organized list</i> | <i>Experiment</i> |
| <i>Draw a picture or diagram</i> | <i>Act it out</i> |
| <i>Look for a pattern</i> | <i>Work backwards</i> |
| <i>Make a table</i> | <i>Use deduction</i> |
| <i>Use a variable</i> | <i>Change your point of view</i> |

3. Carry out the plan

- Carrying out the plan is usually easier than devising the plan
- Be patient – most problems are not solved quickly nor on the first attempt
- If a plan does not work immediately, be persistent
- Do not let yourself get discouraged
- If one strategy isn't working, try a different one

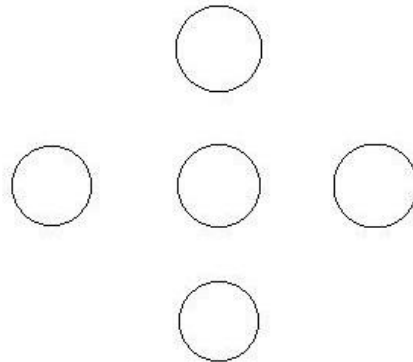
4. Look back (reflect)

- Does your answer make sense? Did you answer all of the questions?
- What did you learn by doing this?
- Could you have done this problem another way – maybe even an easier way?

Problem Solving Strategies – Sample Problems for Students to Apply Different Heuristics

Guess and Check

Put the numbers 2, 3, 4, 5, and 6 in the circles to make the sum across and the sum down equal to 12. Are other solutions possible? List at least two, if possible.



SOLUTION: One possibility
Other solutions possible.
Have students suggest those.

2		
3	4	5
6		

Make an Organized List

List the 4-digit numbers that can be written using each of 1, 3, 5, and 7 once and only once. Which strategy did you use?

SOLUTION:

1357	1735	3517	5137	5713	7315
1375	1753	3571	5173	5731	7351
1537	3157	3715	5317	7135	7513
1573	3175	3751	5371	7153	7531

24 possible 4-digit numbers.

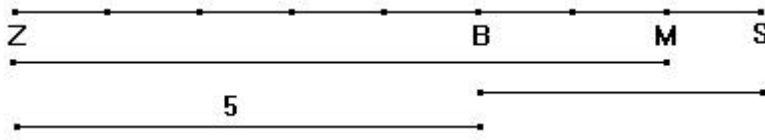
Draw a Diagram

Four friends ran a race:

- Matt finished seven seconds ahead of Ziggy.
- Bailey finished three seconds behind Sam.
- Ziggy finished five seconds behind Bailey.

In what order did the friends finish the race?

SOLUTION:



The order was: Sam, Matt, Bailey, and Ziggy.

Make a Table

Stacey had 32 coins in a jar. Some of the coins were nickels, the others were dimes. The total value of the coins was \$2.80. Find out how many of each coin there were in the jar. What problem solving strategy did you use?

SOLUTION: 8 nickels, 24 dimes

Look for a Pattern

Copy and continue the numerical sequences:

3, 6, 9, 12, _____, _____, _____

27, 23, 19, 15, 11, _____, _____, _____

1, 4, 9, 16, 25, _____, _____, _____

2, 3, 5, 7, 11, 13, _____, _____, _____

SOLUTION:

3, 6, 9, 12, 15, 18, 21

multiples of three

27, 23, 19, 15, 11, 7, 3, -1

subtract 4 from the previous term

1, 4, 9, 16, 25, 36, 49, 64

perfect squares

2, 3, 5, 7, 11, 13, 17, 19, 23

prime numbers

Solve a Simpler Problem

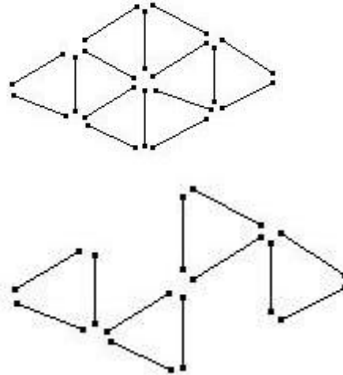
The houses on Market Street are numbered consecutively from 1 to 150. How many house numbers contain at least one digit 4?

SOLUTION: 33 house numbers have at least one digit 4

Act it Out or Use Objects

Sixteen toothpicks are arranged as shown. Remove four toothpicks so that only four congruent triangles remain.

SOLUTION:



Work Backwards

I went into a store and spent half of my money and then \$20 more. I went into a second store and spent half of my money and then \$20 more. Then I had no money left. How much money did I have when I went into the first store?

SOLUTION: \$120 to begin with.

Use Logical Reasoning

Five oranges and a banana cost 87 cents. An orange and five bananas cost 99 cents. What is the total cost of two oranges and two bananas?

SOLUTION: 62 cents for two oranges and two bananas

Brainstorm and Write an Equation

Three pears weigh the same as a quince. A quince weighs as much as eighteen raspberries. How many raspberries weigh the same as a pear?

SOLUTION: Six raspberries weigh the same as one pear.

Reardon, T. Teaching Problem Solving Strategies in the 5-12 Curriculum. Retrieved from <http://www.pdfdrive.net/students-problems-using-problem-solving-strategies-pss-1-e8474067.html>

There's More than One Way to Use a Multiplication Table!

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Rules of Exponents

	Rule	Example
1	$x^1 = x$	$5^1 = 5$
2	$x^0 = 1$	$5^0 = 1$
3	$x^{-1} = \frac{1}{x^1}$	$5^{-1} = \frac{1}{5}$
4	$(x^m)(x^n) = x^{m+n}$	$(x^2)(x^3) = x^{2+3} = x^5$
5	$\frac{x^m}{x^n} = x^{m-n}$	$\frac{x^3}{x^2} = x^{3-2} = x^1$
6	$(x^m)^n = x^{(m)(n)}$	$(x^3)^2 = x^{(3)(2)} = x^6$
7	$(xy)^n = x^ny^n$	$(xy)^3 = x^3y^3$
8	$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$	$\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$
9	$x^{-n} = \frac{1}{x^n}$	$x^{-2} = \frac{1}{x^2}$

Properties of Zero (www.mathsisfun.com/numbers/zero.html)

Property	Example
$a + 0 = a$	$4 + 0 = 4$
$a - 0 = a$	$4 - 0 = 4$
$a \times 0 = 0$	$6 \times 0 = 0$
$0 / a = 0$	$0/3 = 0$
$a / 0 = \text{undefined}$ (dividing by zero is undefined)	$7/0 = \text{undefined}$
$0^a = 0$ (a is positive)	$0^4 = 0$

Symbolic Notation

Sign	Arithmetic	Algebra
= (equal)	. . . And the answer is	Equivalence between two quantities
+	Addition operation	Positive number
-	Subtraction operation	Negative number

Resources from the World Wide Web

Mathematical Reasoning

Annenberg Learner. Courses of study in such areas as algebra, geometry, and real-world mathematics. The Annenberg Foundation provides numerous professional development activities or just the opportunity to review information in specific areas of study.

<http://www.learner.org/index.html>

Florida IPDAE. Lesson plans for both ABE and GED®-level mathematics developed by Florida adult educators. <http://www.floridaipdae.org>

Free Resources for Educational Excellence. Teaching and learning resources from a variety of federal agencies. This portal provides access to free resources. <http://free.ed.gov/index.cfm>

Get the Math. How algebra is used in real-world situations. <http://www.thirteen.org/get-the-math/>

Khan Academy. A library of over 2,600 videos covering everything from arithmetic to physics, finance, and history and 211 practice exercises. <http://www.khanacademy.org/>

The Math Dude. A full video curriculum for the basics of algebra.

http://www.montgomeryschoolsmd.org/departments/itv/MathDude/MD_Downloads.shtml

Media4Math. This site provides you with information/articles of how math is used in the real world. <http://www.media4math.com/MathInTheNews.asp>

PBS Teacher Source. Lesson plans and lots of activities are included in the teacher section of PBS. <http://www.pbs.org/teachers>

Real-World Math. Ideas for how math is used in today's world. <http://www.realworldmath.org/>

Teacher Guide for the TI-30XS MultiView™ Calculator – A guide to assist you in using the new calculator, along with a variety of lesson plans for the classroom.

http://education.ti.com/en/us/guidebook/details/en/62522EB25D284112819FDB8A46F90740/30x_mv_tg

<http://education.ti.com/calculators/downloads/US/Activities/Search/Subject?s=5022&d=1009>

TES. With more than 2.3 million registered online users in over 270 countries and territories, TES provides a wealth of free resources in all academic areas. <http://www.tes.co.uk/>