Teaching Algebraic Reasoning Like a Boss

Information, Resources, and Strategies for the Classroom

WE'RE NOT DONE YET

2024 GED CONFERENCE
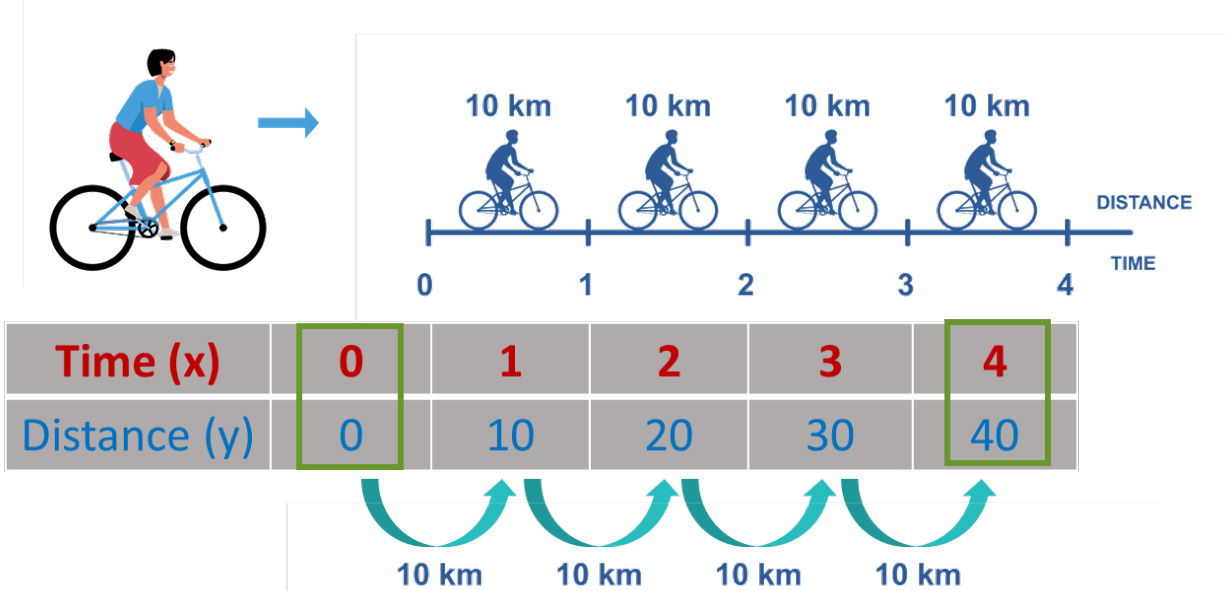
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Recognizing Rate of Change as Slope

A girl is riding a bike at speed of **10 km/hr**.

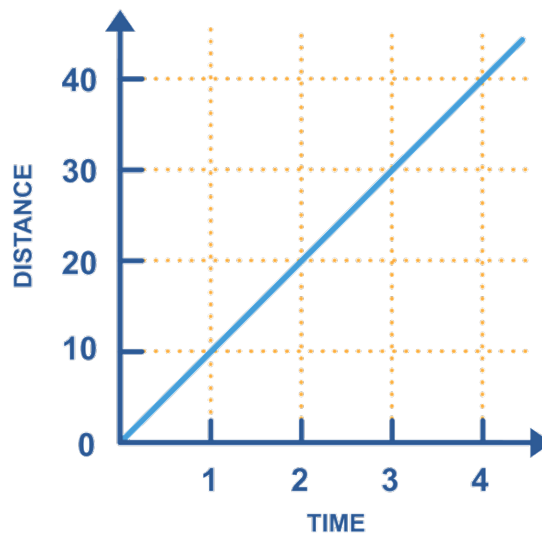| Time (x)	0	1	2	3	4
Distance (y)	0	10	20	30	40

10 km   10 km   10 km   10 km

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{10}{1} = 10 \qquad m = \frac{40 - 0}{4 - 0} = \frac{40}{4} = 10$$

$$m = \frac{\text{rise (change in } y)}{\text{run (change in } x)}$$

$$m = \frac{20}{2} = 10$$

Collaborative Problem-Solving Activity

Fleeing a totalitarian country, pregnant Mia must fight for her life when a violent storm knocks over the container in which she had been hiding overboard. Trapped, Mia gives birth and is forced to survive to save her baby.

Mia and her child are trapped inside a sinking shipping container, slowly filling up with water. She marked and measured the water levels at 0.5-hour intervals using a masking tape and set 5 feet as the maximum water height limit.

She was able to call her husband and give him the different measurements of the water levels up to the moment right before her phone died. Can you help find out how much time rescuers have left to save Mia and her baby?

| Time (h)	Water Height (ft)
2.0 hrs.	1.37 ft.
1.5 hrs.	1.33 ft.
1.0 hrs.	1.29 ft.
0.5 hrs.	1.25 ft.

Finding the Slope of a Line

Table	Equation			Graph
	Slope-Intercept	**Point-Slope**	**Standard**	
Use the slope formula. $$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$	$y = mx + b$	$y - y_1 = m(x - x_1)$	$cx + dy = e$	
Example:	Locate m in the equation.	Locate m in the equation.	Transform equation to slope-intercept form and locate m in the equation.	Locate two points on the graph, then use the slope formula.
	Example:	Example:	Example:	Example:
 x / y 1 / -9 3 / -6 5 / -3	$y = 3x - 4$	$y - 3 = 2x + 8$	$3x + 9y = 4$	$(0,2) \text{ and} (2,3)$
	$y = mx + b$	Factor out the Greatest Common Factor (GCF) to reveal the slope.	$\dfrac{-3x \qquad -3x}{9y = -3x + 4}$	$m = \dfrac{(y_2 - y_1)}{(x_2 - x_1)}$
$m = \dfrac{-3 - (-9)}{5 - 1}$	$m = 3$	$y - 2 = 2(x + 4)$	$\dfrac{9y}{9} = \dfrac{-3x}{9} + \dfrac{4}{9}$	$m = \dfrac{3 - 2}{2 - 0} = \dfrac{1}{2}$
$m = \dfrac{6}{4} = \dfrac{3}{2}$		$m = 2$	$y = \dfrac{-3}{9}x + \dfrac{4}{9}$	
			$m = \dfrac{-3}{9}$	

Solving Systems by Graphing: Cost and Revenue Problem

Instructions: Solve the problem below by graphing.

Gina sells customs stickers on Esty. The cost to make one sticker, including material and labor is $3.00. She also pays a fixed cost of $10 for sticker paper subscription. If she wants to sell her custom stickers for $5.00 each, how many stickers does she need to sell to break even?
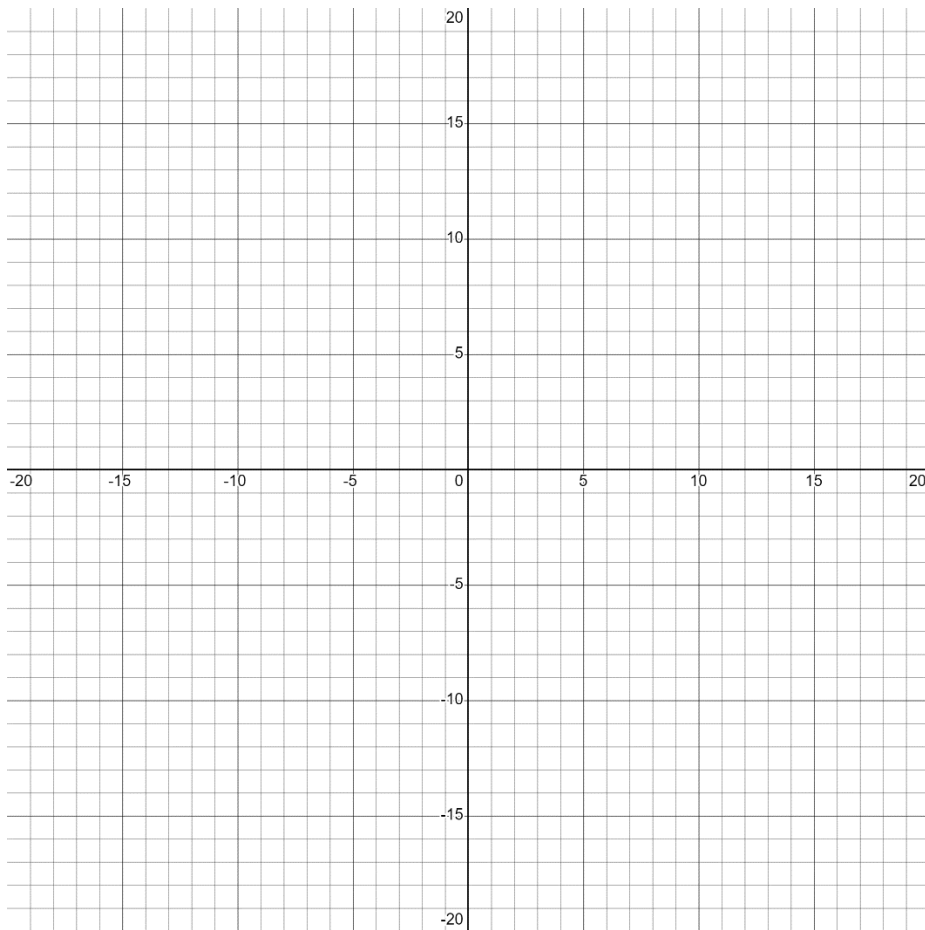
W.I.S.E. Method: Solving Systems by Substitution

1. Write the equation.
2. Isolate the unknown.
3. Substitute the new equation and solve.
4. Evaluate for the remaining unknown.

Instructions: Solve the problem below by substitution.

Coin Problem: Alex has a collection of nickels and dimes. The total value of the coins is $2.30. If he has a total of 30 coins, how many nickels and how many dimes does Alex have?

Tips for Solving Systems by Linear Combination

- Examine the equations first.
- Check if adding the equations will eliminate a variable.
- Look for multiples or factors.
- Decide which variable to eliminate.
- Try to avoid fractions/decimals.
- Try to avoid negatives.

Instructions: Determine the fastest solution method for solving the following systems of linear equations.

1. $3x + 6y = 12$
   $5x - 8y = 2$
   Fastest/Best solution method: _____

2. $y = 5x + 2$
   $y = -x + 8$
   Fastest/Best solution method: _____

3. $-4x + 7y = 3$
   $2x - 2y = 2$
   Fastest/Best solution method: _____

4. $-2x - 3y = 60$
   $x = 2y$
   Fastest/Best solution method: _____

5. $-x + 2y = 7$
   $-2x + 4y = 4$
   Fastest/Best solution method: _____

6. $y = -3x + 5$
   $x = y$
   Fastest/Best solution method: _____

Simple Trick for Solving Systems of Linear Equations

Solve the system of linear equations below using the simple trick.

$3x + 6y = 12$

$5x - 8y = 2$

What is a Quadratic Equation?

The shape of the arch is formed by graphing of the equation.

$$y = -(x + 10)(x - 10)$$

- What do you notice about the numbers in the equation and the graph?

- What do you notice about the shape of the graph?

- What do you think will happen if we removed the negative sign in front of the equation?

- How do you think the height of the arch is obtained?
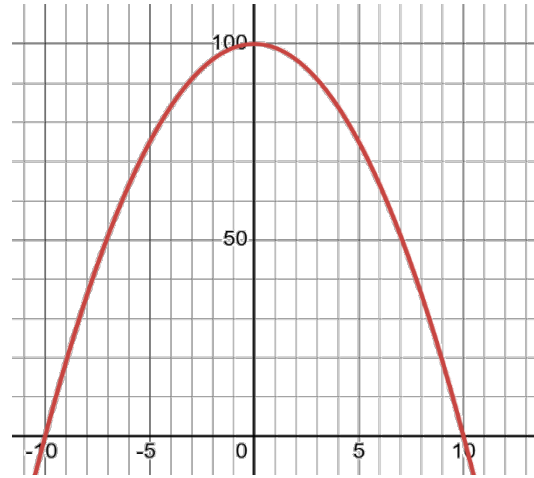
Key Features of Parabolic Graphs

Graph the line representing the Multiples of 4 table below.
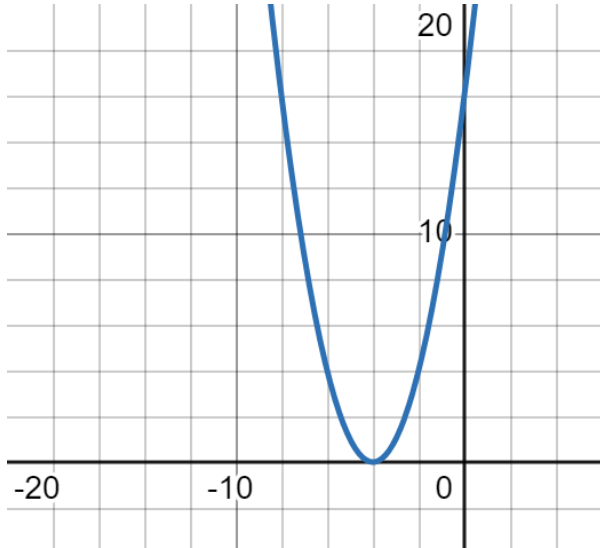
$$y = (x + 4)(x + 4)$$

Facing: _____

Root(s): _____

Vertex: _____
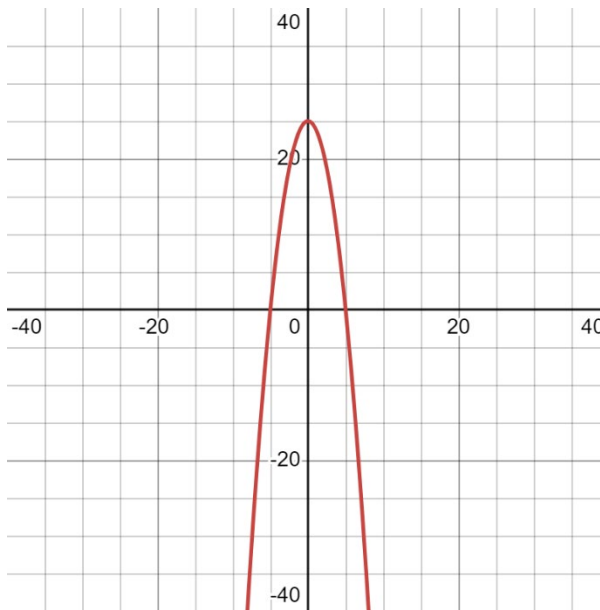
$$y = -(x - 5)(x + 5)$$

Facing: _____

Root(s): _____

Vertex: _____
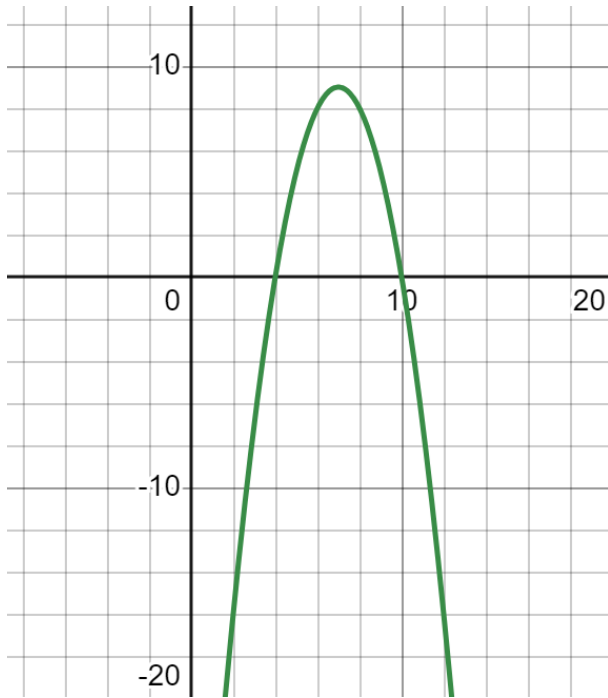
$$y = -(x - 4)(x - 10)$$

Facing: _____

Root(s): _____

Vertex: _____

Forms of Quadratic Equations

1. **Factored Form**

$$y = a(x + r_1)(x + r_2)$$

2. **Standard Form**

$$y = ax + bx + c$$

3. **Vertex Form**

$$y = a(x - h)^2 + k$$

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Instructions: Use the quadratic formula to find the roots of the quadratic equations below.

1. $y = x^2 - 8x + 15$

2. $y = x^2 - 8x + 16$

3. $y = x^2 + 2x + 5$

Solving Quadratic Equations by Factoring

Solve the following quadratic equations by factoring. Show your work.

1

$y = x^2 + 6x + 5$

2

$y = -3x^2 - 6x + 144$

3

$$y = -2x^2 + 10x$$

Solving Quadratic Equations by Completing the Square

Solve the following quadratic equations by completing the square. Show your work.

1. $y = x^2 + 6x - 7$

2. $y = x^2 + 6x + 5$

Real-World Problems Leading to Quadratic Equations

Solve the following word problems.

1. **Area Problem:** The width of a rectangle is 6 meters less than its length. The area is 72 square meters. Find the dimensions of the rectangle.

2. **Number Problems:** The product of two positive consecutive odd integers is 99. Find the integers.

Vocabulary of Polynomials

• Term	• Additive Inverse
• Similar Terms	• Model
• Polynomial	• Combine
• Monomial	• Factor
• Binomial	• Distribute
• Trinomial	• Evaluate
• Zero Pair	• Expand
• Integer	• Substitute

Algebra Tiles Template

x^2	x^2	x	x	x	x
x^2	x^2	x	x	x	x
x	x	1	1	1	1
x	x	1	1	1	1
x	x	1	1	1	1
x	x	1	1	1	1

Algebra Tiles Equation Mat
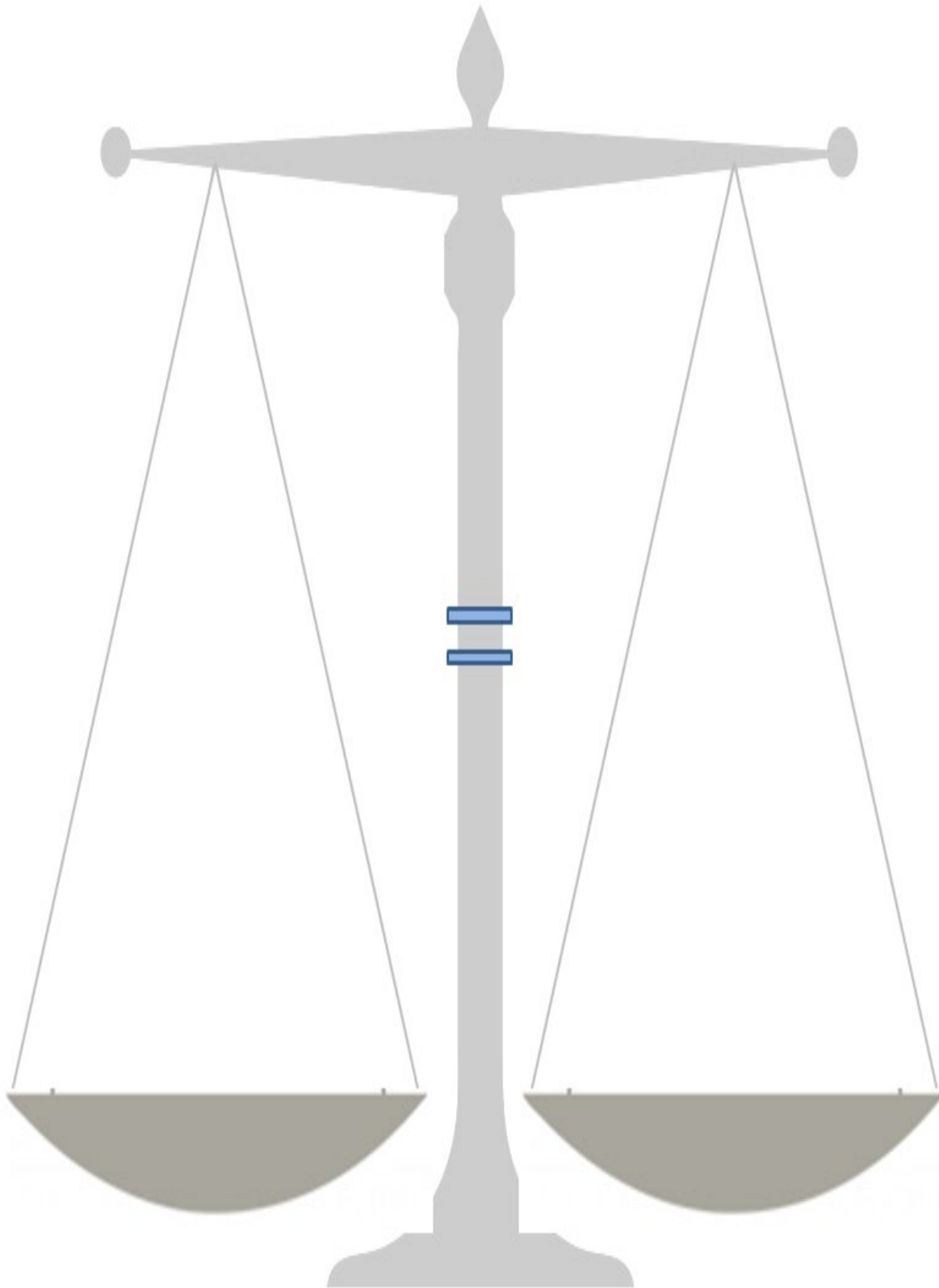
Algebra Tiles Product Mat

X
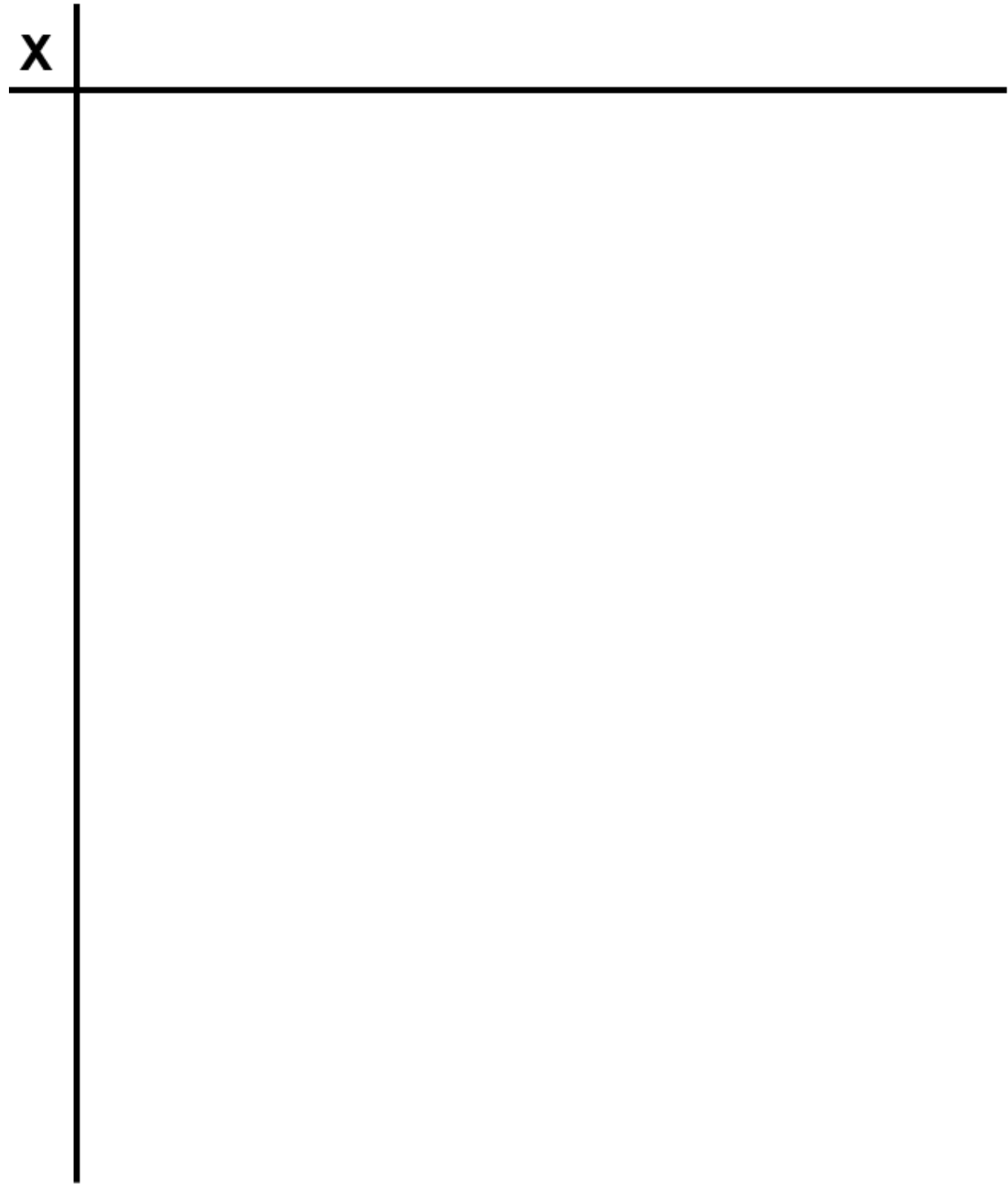
Operations on Polynomials

Perform the indicated operation.

$$
\begin{array}{r}
6x^5 - 10x^2 + x - 45 \\
+ \qquad 13x^2 - 9x + 4 \\
\hline
\end{array}
$$

$$
\begin{array}{r}
5x^4 - 9x^2 + x - 3 \\
- \qquad 2x^2 + x - 1 \\
\hline
\end{array}
$$

$$4x(x^2 - 6x + 2) =$$

$$(3x - 5)(6x - 10) =$$

Evaluating Polynomials

Solve the problem.

A volleyball is tossed vertically upwards with an initial velocity of 40 ft/s from a height of 6 feet. If height after t seconds is represented by $h = -16t^2 + 40t + 6$. What is the height of the volleyball after 2 seconds?

Factoring Polynomials

Factor the following polynomials.

$$3x + 9$$

$$4n^3 - 2n^2 + 8n$$

$$a^2 - 2x - 15$$

Dividing Factorable Polynomials

Perform the indicated operation.

$$\frac{2x^2 + 12x + 16}{2x + 4}$$

$$\frac{n^2 - 6x + 9}{x^2 - 9}$$

Solving Real-World Problems Leading to Polynomials

Solve the problem below.

A tree is supported by a wire anchored to the ground 5 feet away from its base. The wire is 1 foot longer than the height that it reaches on the tree. How long is the wire?

Resources from the World Wide Web

Assessment Guide for Educators: Mathematical Reasoning
https://ged.com/wp-content/uploads/assessment_guide_for_educators_math.pdf

Coordinate Plane Graph
https://www.math-aids.com/Graph_Paper/Coordinate_Plane_Graph_Paper.html

Desmos Graphing Calculator
https://www.desmos.com/calculator

Didax Algebra Tiles
https://www.didax.com/apps/algebra-tiles/

Forms of Quadratics: Explanations, Tips, and Examples
https://www.albert.io/blog/forms-of-quadratics/

GED® High Impact Indicators
https://www.ged.com/wp-content/uploads/High_Impact_Indicators.pdf

GED® Performance Level Descriptors Level 1
https://www.ged.com/wp-content/uploads/pld_math_official_test_below_passing.pdf

GED® Performance Level Descriptors Level 2
https://www.ged.com/wp-content/uploads/pld_math_official_test_passing.pdf

GED® Performance Level Descriptors Level 3
https://www.ged.com/wp-content/uploads/pld_math_official_test_college_ready.pdf

GED® Performance Level Descriptors Level 4
https://www.ged.com/wp-content/uploads/pld_math_official_test_college_ready_credit.pdf

GED® Tips for Attaining High School Equivalency
https://www.ged.com/wp-content/uploads/Tips_Math_Attaining_HS_Equiv.pdf

Mathster Graph Paper Generator
https://mathster.com/graphpaper/graphpaperjs/

NCTM Illuminations Algebra Tiles
https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Algebra-Tiles/

Polynomial Equation Word Problems
https://www.onlinemathlearning.com/polynomial-equation-word-problem.html

Quadratic Word Problems
https://andymath.com/quadratic-word-problems/

Roots of Quadratic Equation
https://www.cuemath.com/algebra/roots-of-quadratic-equation/

Short Trick to Quadratic Equations with Large Coefficients
https://www.youtube.com/shorts/E-FAgYelQgg

The Quadratic Formula Explained
https://www.purplemath.com/modules/quadform.htm

Virtual Algebra Tiles
https://mathsbot.com/manipulatives/tiles